

I GOALS AND OBJECTIVES

While still a relatively new graduate student, I once remarked to my advisor, Jim Cushing, that I still didn't understand quantum mechanics. To this he promptly replied: "You'll spend the rest of your life trying to understand quantum mechanics!"

As one of our most fundamental physical theories, quantum mechanics is immensely important, but it is also exceedingly abstract and difficult. In the undergraduate physics curriculum, 'real' quantum courses are often preceded by a modern physics course, but these are typically rather conceptual; the highly abstract mathematical and physical concepts of quantum mechanics remain largely untouched. There is thus a rather large gap to be bridged, and students in advanced courses often find they must solve problems and learn new applications even while the framework of quantum mechanics remains unclear.

Einstein said that physics should be made as simple as possible, but no simpler. **A new book, *Essential Quantum Mechanics (EQM)*, will accurately render the fundamental structure of quantum mechanics as simple as possible, but no simpler.** EQM will provide a concise (150-200 pages) yet accurate bridge to 'real' quantum mechanics by focusing on the theory's physical concepts and mathematical framework.

EQM will be aimed primarily at undergraduates about to take, or taking, their first quantum course. A knowledge of elementary calculus and complex variables should provide ample mathematical background. (A condensed discussion of these topics will be presented in Appendix 1 of EQM.) Topics to be covered are laid out in detail in Section II C. Notable by their absence are many standard topics, such as perturbation theory, scattering, and the Hydrogen atom. This is in keeping with one of the central motivating ideas of EQM: that to properly understand the many and varied applications of quantum mechanics, one must first properly

understand its overall structure. EQM is intended in large part to fill a real void in the existing literature, by presenting that structure in a particularly clear and concise manner – as simply as possible, but no simpler.

II PROJECT PLAN

A *The Vision of “Essential Quantum Mechanics”*

My interest in writing EQM grew out of my experience in graduate school, where I found myself ill-prepared for my graduate quantum mechanics courses. After surviving that trial by fire, I wondered why the fundamentals were not more clearly explained. And I concluded that, in part, it was due to the textbook authors’ interest in providing a rigorous, general, and comprehensive treatment – a worthy goal, but one that may not best serve all readers.

EQM will take a new tack, stressing a clear and accurate exposition of fundamentals, while leaving rigorous, expansive treatments to the advanced textbooks. A variety of tools will be used to make EQM especially useful for the student. By focusing on fundamentals, such as superposition and time evolution, they may be developed in a more detailed and explanatory style than in advanced texts. Variations in terminology and notation that the student may encounter will be clarified (*e.g.*, the terms quantum state, state vector, and wavefunction). In addition, rather than just stating what is right, I will discuss what is wrong. For example, although position-space and momentum-space are standard topics, students often fail to realize that there is but one quantum state, which may be cast into various representations. Such potential stumbling blocks will be explicitly pointed out and explained.

David Park (Park, 1964, p. vi) wrote of intelligent physicists who, despite having learned to solve quantum-mechanical problems, had not acquired a proper understanding. In

consequence, "...having in the course of time lost hold of the computational devices, they have lost all, and no instinct remains to guide them." Because physical instinct is born of physical understanding, providing a clear interpretation of the quantum formalism is a particularly important task of any heuristic approach to quantum mechanics. I am well suited to carry out this difficult task, in part because of my research in foundations of quantum mechanics. I will adopt and emphasize the statistical interpretation – a very useful and lucid means to obtain an understanding of how quantum mechanics works in actual practice (Ballentine, 1970). Because this approach is really a broad framework rather than an interpretation *per se*, it is easily ‘imported’ into other approaches as the student may find necessary.

B *Comparison with other books*

Comprehensive undergraduate quantum texts, such as (Liboff, 2003) and (Griffiths, 1995), are typically 400-500 pages in length, and make very considerable mathematical and technical demands on their users. EQM is better compared with short undergraduate introductions, such as (Martin, 1981) and (Rae, 1986). Although these books are fairly concise (some 250 pages), they are still written at a level similar to comprehensive quantum texts. Also like other texts, and unlike EQM, they concentrate on expansion and application of the quantum-mechanical structure, rather than on its development and elucidation. The mathematical level is fairly high, and could prove daunting for many undergraduates.

Modern physics texts (*e.g.*, Bernstein *et al.*, 2000) generally take a historical approach, discussing topics such as the Bohr atom and the Compton effect. Formalism is minimized and description emphasized – for example, by focusing on wavefunctions and energy eigenstates

rather than general quantum states. This illustrates both the gulf that students must bridge in their first real quantum course, and how EQM will differ greatly from modern physics texts.

Another useful comparison is with (Stevens, 1995). Chapter 4 of this compact book is a very concise (34 pages), self-contained exposition of the fundamentals of quantum mechanics. Although nicely done, the discussion is quite sophisticated and very condensed, and clearly serves a different audience than will EQM.

One other text of interest is (Ballentine, 1998). True to its title – *Quantum Mechanics, A Modern Development* – Ballentine’s book *is* modern: many topics of current interest are discussed, Dirac notation is used early and often, the statistical interpretation is adopted, and foundational issues are treated carefully and diligently. But Ballentine’s book is unmistakably a graduate text, and a challenging one at that. I find much of value in Ballentine’s approach – it has, in part, inspired EQM – but it certainly will not duplicate it.

There are now many popularizations of quantum mechanics available. Two examples are (Albert, 1992) and (Styer, 2000). (I was a pre-publication reviewer for Styer.) The mathematical level of such books varies from very low to essentially nonexistent. Popularizations such as these are intended not to expound quantum mechanics as used by working scientists, but rather to discuss ‘quantum weirdness,’ such as Bell’s theorem and nonlocality, and the measurement problem, in terms palatable to interested nonscientists. Thus, they serve a much different purpose than does EQM.

Textbooks, popularizations, and *Essential Quantum Mechanics* – all have legitimate but distinct goals. EQM will complement, but not duplicate, books such as those described above.

C *Preliminary Table of Contents*

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Undergraduate science and engineering students constitute the core constituency at which this project is aimed. And, as one research group has suggested, “there is an increasing need for quantum mechanics to be understood by professionals who will never be researchers” (Johnston *et al.*, 1998). EQM could serve as a supplement for a standard quantum text, or as the primary text for a ‘mini-course’. A condensed course based on EQM will better prepare physics students for advanced courses, but will also be valuable for chemistry and electrical engineering students, who may not otherwise take a quantum course, even though quantum mechanics plays a central role in their fields of study.

Apart from its core readership, EQM will be of interest to a variety of readers such as biologists, philosophers, and other thoughtful people who are fascinated by quantum mechanics, but find the popularizations too simplistic, and the textbooks too advanced and comprehensive (and expensive!). EQM will provide a means for such people to gain a foothold on ‘real’ quantum mechanics.

The goal of *Essential Quantum Mechanics* is to fill an important lacuna by accurately presenting the essential structure of quantum mechanics in a uniquely concise and explanatory manner.