Homework set #5 (assigned 13 April, due 22 April)

Instructions. As always, these homework questions have many parts to them. Make sure you read the entire question and answer each and every part! You do not get any credit for problems or parts of problems that you do not attempt. You should make sure that in all cases your units agree. That is, if you are using meters, seconds, and grams for your calculations, make sure you don't accidentally have any kilometers, years, or kilograms (or, heaven forbid, pounds) in your calculations or solutions.

1) The nearest star to the Earth is Proxima Centauri, about 4.2 light years distant (4.2 light years = 4×10^{13} km = 1.3 parsecs). A typical very nearby star might be 20 parsecs distant. Escape velocity for our Solar System – that is, the velocity that a comet/asteroid/spaceship must have in order to leave our Solar System – is something like 600 km/sec. What is the travel time, assuming escape velocity, between a typical very nearby star and the Earth? Your answer should be in years.

Life is thought to have originated very early on Earth, probably as early as 3.8 billion years ago (recall that the age of the Earth is around 4.5 billion years). The particularly violent period of the Earth's history called Late Heavy Bombardment ended around 3.9 billion years ago. One criticism/concern about the origin of life on Earth is that perhaps that 100 million years wouldn't be enough time for life to originate here on Earth. The counter-proposal is that life could have originated elsewhere in the Universe and traveled to Earth.

Using the travel time you calculated above, does this proposed solution help at all?

The galaxy is around 100 kiloparsecs across (1 kiloparsec = 1000 parsecs). Assuming life originated around a Sun-like star, from how far across the galaxy could a panspermic life-form travel to the Earth if the oldest Sun-like star in the galaxy is around 10 billion years old? How much time could you allow for life to originate if it originated at the birth of a 10 billion year old star on the opposite side of the galaxy?

The typical density of very nearby Sun-like stars might be around 0.01 Sun-like stars per cubic parsec (think of a volume in space; the volume of a sphere is equal to $\frac{4}{3}\pi r^3$ where r is the radius of the sphere). How many Sun-like stars are there within the typical very nearby volume (of, say, radius 20 pc)? How about in the whole galaxy (assuming that the space density of Sun-like stars is constant throughout, which it almost certainly isn't)? You can assume the galaxy is a sphere (which, again, it isn't). If life had to originate around one of these Sun-like stars in order to travel panspermically to the early Earth, how many eligible Sun-like stars are there from which life could have begun its travels?

Now imagine that you are riding along on a panspermically-rich comet or asteroid that has made its way from another planetary system to our Solar System. You start to approach our Solar System (radius around 50 AU, where 1 AU is 1 Astronomical Unit, which is equal to 1.5×10^8 km); imagine that you have a "top-down" view, where you are looking down on the planets orbiting around the Sun. What is the probability that out of the entire Solar System's area (as seen from your perspective), you will land on Earth? (The planetary radius of Earth is around 6700 km.) It will certainly help you to figure out this problem if you *draw a picture.*

Lastly, how likely do you think this all is? By all, I mean all of the steps you have considered above here. You can calculate this numerically but you do not have to – you can answer in words/thoughts/gut feelings instead.

2) This is the hardest math problem I have given you this semester. Think of it as an opportunity to stretch your math wings. This problem is not really all that hard, but is a little more involved than other problems I have given you.

We have talked about habitable zones in class. It is actually pretty straightforward to calculate the location of the habitable zone, both as a function of stellar temperature (that is, for different *stellar spectral types*) and distance, and as a function of time in the Solar System (remember the faint early Sun paradox and the continuous habitable zone).

The temperature of a planet (T_{pl}) is given by the following:

$$T_{pl}^4 = \frac{T_\star^4 R_\star^2 (1 - A)}{4a^2} \tag{1}$$

where $T_\star$ is the temperature of the star heating the planet, $R_\star$ is the radius of the star, A is the albedo (reflectivity) of the planet, and a is the orbital distance of the planet.

Calculate the location of the habitable zone as a function of stellar temperature and distance. You should be able to reproduce the figure we saw in lecture. However, of course I want you to actually do the calculations and not just copy the figure from the lecture. Make a plot (stellar temperature versus distance from the star) that shows the location of the habitable zone. Mark the stellar spectral types on your plot as well. You can assume a planetary albedo of 0.5 (that is, 50% reflective; the Earth's albedo is more like 28%, but that's okay). You can also assume that stellar radii and stellar temperatures are related in this way:

$$\left(\frac{R}{R_\odot}\right) = \left(\frac{T}{T_\odot}\right)^{1.3} \tag{2}$$

which is not really exactly true but is good enough for us to use in this problem ($R_\odot$ and $T_\odot$ are the radius and temperature of the Sun). You should also ignore all atmospheric effects. This is obviously a terrible assumption, as a planet in the habitable zone with no atmosphere isn't going to be much of a habitable planet, but it makes the problem much easier.

You can also calculate the evolution of the location of the habitable zone in our Solar System over time. The luminosity of the Sun as a function of time is given by this (Gough 1981):

$$L_\odot(t) = \left[1 + \frac{2}{5}\left(1 - \frac{t}{t_\odot}\right)\right]^{-1} \times (L_{\odot,today}) \tag{3}$$

where t is time (in billions of years) and $t_\odot$ is the current age of the Sun, 4.5 billion years; and $L_{\odot,today}$ is the luminosity of the Sun today (3.9×10^{26} J/sec). Using this equation, make a plot (distance versus time) that shows the location of the habitable zone in our Solar System over time.

You might also find it useful to know that the luminosity of a star – the amount of energy the star emits per second – is given by this equation:

$$L_\star = 4\pi R_\star^2 \sigma T_\star^4 \tag{4}$$

where $L_\star$ is the luminosity (units: Joules per second); $R_\star$ is the radius of the star (units: meters); and $T_\star$ is the temperature of the star (in Kelvins). Sigma (σ) is the Stefan-Boltzmann constant (5.67×10^{-8} J/m^2/K^4/second).

On your plot showing the evolution of our Solar System's habitable zone with time, mark (shade in) the location of the *continuous habitable zone*. Also mark the Earth. Briefly (one or two sentences) comment. How would including a planetary atmosphere affect your answer?

3) In class we will talk about the Drake Equation – see your textbook for an introduction. In class I showed you XKCD's version, the Flake Equation (`http://xkcd.com/718/`).

Create your own version of the Drake/Flake Equation. Do anything you want! The Steak Equation? (Number of people eating meat at any given time.) The Break Equation? (Number of broken arms across the world at any given time.) The Jake Equation? (Number of NAU students named Jake.) Have a ball – do whatever you like. Best entries get posted to the entire class. Make sure you have at least five terms in your equation, and give a little description of it to make sure I can understand it.