Solutions: Homework set #4 (assigned 25 March, due 8 April)

There were a lot of different aspects to this assignment, from deep computations to subtle thinking. I tried to make it somewhat of a synergy of various things we have learned in the course (amino acids and late heavy bombardment, for example). And I know it was hard, but I hope you learned some cool stuff from it. I encourage you to read this solution set thoroughly, both to see how I did these problems but also to see what kinds of things can be learned by applying geometry and algebra to problems in the Solar System.

The same comments as always apply for this homework: you must show all your work, you must be careful with your units, you must answer all parts of all questions, and so on. Come talk to me if you're not certain what I've done, or why.

Let me also emphasize that I don't expect that any of you turned in homeworks that looked like this – this is way more rigorous and complete than I expected you to do. I have written as complete a solution set as I could to make it as clear as possible what I did. As always, please ask if what I did is not clear. I would be happy to review these solutions and your work with you.

The main problem many people had with their homeworks was unit conversions. You must be very careful with converting centimeters to kilometers and especially m^3 to km^3 (for example). You know that 1000 m = 1 km. Therefore, 1 km^3 must equal $(1000 \text{ m})^3$, or 10^9 m. Checking units is an easy thing to do and I encourage you always to do it. Bottom line: remember to be careful and check to make sure your units agree, cancel out, and make sense. I would be more than happy to go over various units issues with you if you like – come see me.

1) The heat flow from Io is 2.5 W/m^2 which is 2.5 $J/m^2/sec$. The radius of Io is 1800 km, so the total heat energy escaping from Io per second is 2.5 $J/m^2/sec \times 4\pi r^2$ which is around 10^{14} J/s. The volume of the crust is (using a depth of 10 km and the equation for the volume of a shell) around 4.1×10^{17} m^3. Using a density of 3000 kg/m^3, we get a mass of 1.2×10^{21} kg.

The latent heat of melting tells us how much heat (how many Joules) it takes to melt 1 kg of material. Since I know the mass of the whole of Io's crust, I can figure out the latent heat of melting of Io's crust: 5×10^5 J/kg $\times$ 1.2×10^{21} kg gives 6×10^{26} J. Since we know that Io's heat flux is 10^{14} J/s, we can figure out that it would take 6×10^{12} seconds to melt all of Io's crust. This is about 200,000 years. In terms of the age of the Solar System and the frequency of impacts, this is a very, very short time: the Earth gets hit by one dinosaur-killing-sized comet every 100 million years (you'll see this in problem 2) – Io's resurfacing timescale is extremely short compared to this. No matter when an impact happens on Io, the surface gets remelted very quickly, and all evidence of impacts gets erased through the melting. This is why we see no impact craters on Io.

Europa's heat flow is about 100 times smaller, so about 0.025 W/m^2. The volume of the water ice layer is found by

$$V_{water} = V_{Europa} - V_{not\ water} \tag{1}$$

where $V_{not\ water}$ is the volume of the rest of the moon that isn't water. If you used the thin shell formula here, that's not technically correct (because 100 km is not much, much less than 1600 km, the radius of Europa), but not a big deal (I didn't take off a lot of points). V_{Europa} is simply $\frac{4}{3}\pi R_{Europa}^3$ which is 1.6×10^{19} m^3. $V_{not\ water}$ is found by $\frac{4}{3}\pi R_{not\ water}^3$ where $R_{not\ water}$ is R_{Europa} minus 100 km. Thus, $V_{not\ water}$ is 1.33×10^{19} m^3. Therefore, the volume of water is around 2.7×10^{18} m^3. The mass of this water ice layer is 2.7×10^{21} kg (using a density of 1000 kg/m^3). So the total heat needed to melt this water ice layer is the latent heat of melting for water ice times the mass of the water ice layer: 3.4×10^5 J/kg $\times$ 2.7×10^{21} kg which is 9.2×10^{26} J. Europa's total heat flow per second is $4\pi r^2 \times 0.025$ $J/m^2/s$ which is 8×10^{11} J/s (Europa's radius is around 1600 km). Dividing the total heat needed by the heat flow per second gives us 10^{15} seconds, or around 35 million years. Europa should therefore probably have some craters, but not a whole lot: this is still a pretty short resurfacing time compared to the age of the Solar System. In fact, as you know, that is exactly what is observed: Europa has some impact craters, but compared to the Moon, for example, Europa still has relatively few craters.

2) This question asks you to calculate the mass of CO_2 in the Earth's atmosphere and in Venus' atmosphere, and to estimate what percentage of the total carbon reservoir on each of these planets the atmospheric amount represents. There are a few different ways to do this problem; here's one way.

First, how much CO_2 is in the Earth's atmosphere? We can start by finding the mass of the Earth's atmosphere. The Earth's atmosphere is approximately a shell; the volume of a spherical shell is given by $4\pi r^2 \Delta r$, where r is the radius of the sphere and Δr is the thickness of the thin shell. Here, r is the radius of the Earth, and we can use a thickness equal to the scale height, 10 km. So we get a volume of 5×10^9 km^3 or 5×10^{18} m^3. Since the density of air here on Earth is around 1 kg/m^3, the mass of the Earth's atmosphere is around 5×10^{18} kg. So if CO_2 is 0.035% (350 parts per million), then the mass of CO_2 in the atmosphere is roughly 2×10^{15} kg. It only makes a small difference if you assume parts per million volume or parts per million mass.

CO_2 is made of carbon and oxygen. The atomic mass of carbon is 12 (number of protons plus number of neutrons) and the atomic mass of oxygen is 16, so the atomic mass of CO_2 is 44. Thus, 12/44 of the CO_2 mass is carbon, or 5.5×10^{14} kg.

Now if Venus' surface pressure is 90 bars, that means it is 90 times greater than the Earth's surface pressure. Pressure is force per area: $P = F/A$. We can write a ratio:

$$\frac{P_V}{P_E} = \frac{F_V A_E}{A_V F_E} = 90 \tag{2}$$

where the V subscript means Venus and E means Earth. Remember that the force we are talking about here is the weight of the atmosphere, which is the force of gravity on the atmosphere:

$$F_E \ (atmosphere) = \frac{G M_E m_{E,atm}}{R_E^2} \tag{3}$$

which is just the Universal Law of Gravitation and where $m_{E,atm}$ is the mass of the Earth's atmosphere. I plug equation 3 for Venus and for Earth into the ratio in equation 2 and do some simplifying:

$$\frac{P_V}{P_E} = \frac{F_V A_E}{A_V F_E} = \frac{4\pi R_E^2 F_V}{4\pi R_V^2 F_E} = \frac{R_E^2 G M_V m_{V,atm} R_E^2}{R_V^2 R_V^2 G M_E m_{E,atm}} = \frac{M_V m_{V,atm} R_E^4}{M_E m_{E,atm} R_V^4} \tag{4}$$

where I have substituted the surface areas of Venus and Earth and simplified. Now I rearrange equation 4 to solve for the mass of Venus' atmosphere:

$$m_{V,atm} = m_{E,atm} \left(\frac{P_V}{P_E}\right) \left(\frac{R_V}{R_E}\right)^4 \left(\frac{M_E}{M_V}\right) \tag{5}$$

which is something we can solve: we know the mass of the Earth's atmosphere; the ratio of the pressures (90 bar/1 bar = 90); the radii of the planets; and the masses of the planets. When you plug in the correct numbers (Earth atmosphere mass from above; radius and mass of Earth are 6400 km and 6×10^{24} kg; radius and mass of Venus are 6050 km and 4.9×10^{24} kg), you find that the mass of Venus' atmosphere is 4.4×10^{20} kg. If 96% of that is CO_2, then the CO_2 mass is around 4.2×10^{20} kg. The total amount of carbon in Venus' atmosphere is therefore around 10^{20} kg.

On the Earth, something like 0.00055%, or 1 part in 200,000, of the total carbon on the planet is in the atmosphere. The vast majority (over 99%) of carbon on the Earth is found in carbonate rocks, like limestone. On Venus, however, if you assume that the total carbon reservoir is the same as the Earth's total carbon reservoir, then it looks like all of the carbon on the planet must be in CO_2 in the atmosphere! The conclusion you could draw from this calculation is that there aren't any carbonate (CO_2-bearing) rocks on Venus, something which is suspected but certainly not known currently.

3) Let's take this typical comet. Its volume is 4×10^3 km^3 (using $V = \frac{4}{3}\pi r^3$), or 4×10^{12} m^3. Its total mass is therefore 8×10^{15} kg (density is mass/volume so mass is density times volume). If we assume that 50% of the comet's mass is water, the total water mass is 4×10^{15} kg. To deliver the Earth's entire water supply therefore would require 250,000 such comets (total water mass divided by water mass per comet gives total number of comets needed). Over the period of late heavy bombardment (4.5–3.9 billion years ago), this works out to be one comet every 2400 years. In other words, 2400 years went by between comet impacts (on average), during late heavy bombardment. Still not that frequent, you're thinking, right? Well, a 10 km body is the size of the impactor that wiped out the dinosaurs. Imagine that every 2400 years (instead of every 100 million years as it is now). We know it is 100 million years between 10 km-sized comets now because the influx rate is around 10^{-8} comets per year. You can simply turn this upside down to find 10^8 years per comet, or 100 million years between each comet impact.

Now we assume that amino acids are 0.0003% of the mass of comets, so the total influx of amino acids over that period is

$$0.000003 \times 250,000 \text{ comets} \times 8 \times 10^{15} \text{ kg} = 6 \times 10^{15} \text{ kg of amino acids} \tag{6}$$

which is 6 billion metric tonnes of amino acids. The surface area of the Earth is $4\pi R_{Earth}^2$ which is 5×10^{14} m^2. We know that 6×10^{23} amino acids (glycine) have a mass of 75 g or 0.075 kg. The amino acid mass of 6×10^{15} kg is therefore 4.8×10^{40} glycine molecules. Spread these over the entire surface of the Earth and you get around 10^{26} amino acids per square meter. If each amino acid is 10^{-9} meter in size, then you could lay out $10^9 \times 10^9$ amino acids side by side in one square meter in a layer which is one amino acid tall: this is 10^{18} amino acids in a layer one amino acid tall, per square meter. We have on the early Earth 10^{26} amino acids per square meter, which means we need to have 10^8 layers of amino acids each of which is 1 amino acid tall. A set of 10^8 amino acid layers when each layer is 10^{-9} m thick is a total stack which is 0.1 m tall, or 10 centimeters. That's a pretty thick sludge of amino acids! This implies that there may have been no shortage of amino acids arriving from space for use in prebiotic chemistry – the problem would instead become whether or not these amino acids could be used efficiently before being destroyed through high temperatures or other environmental conditions.

4) Obviously, there is no "right" answer for this question – most people did a fine job of choosing an interesting article, summarizing it, and connecting it to astrobiology. If you failed to connect your article to astrobiology, or if you failed to provide a complete citation for your article, you lost some points.