Solutions: Homework set #3 (assigned 23 February, due 4 March)

General comments. Overall, I was pretty pleased with the work on this assignment. Many people did quite well, especially on what I thought were the harder problems.

Here is a general comment for questions that require word (as opposed to calculation) answers: Don't put down all kinds of irrelevant material just because you know it or read it. I just want the answers to the questions. **No brain dump!**

And of course, it is never sufficient to just look things up on the web. You need to always do the calculations I have asked you to do.

Also: Make sure you answer the "why" part of the questions. Careful about circular logic and causality problems (many people wrote that 'Jupiter is gas because it is a gas giant planet' – no good).

1) Newton's second law is $F = m \times a$. Your weight is really a force; your weight in pounds is equal to your mass in slugs times the acceleration due to gravity (in English units). So $m = F/a$. If I weigh around 160 pounds and $a = 32$ ft/sec^2 then (a) my mass is 5 slugs. Using the conversion (1 slug = 14.6 kg), I find that (b) my mass is 73 kg. My mass (c) 1000 km above the Earth's surface is still 73 kg – it is always the same. But to find my weight 1000 km above the Earth's surface, I have to use the universal law of gravitation:

$$F = \frac{Gm_1 m_2}{r^2} \tag{1}$$

Here, G is big G (always the same: 6.67×10^{-11} m^3/sec^2/kg), m_1 is mass of the Earth, m_2 is my mass, and r is the distance from the center of my mass to the center of the Earth's mass. In this case, r is therefore equal to the radius of the Earth plus 1000 km, so 7400 km total. We can plug all this stuff in to the equation and find that (d) my weight 1000 km above the Earth's surface is 533 N (Newtons is the unit of force in the metric system.). [Aside: What does that mean, 533 N? Well, you can figure out what my weight is, in Newtons, at the Earth's surface, by using $r = 6400$ km. With that, you get 713 N. So 1000 km above the Earth's surface, I weigh around 75% of what I weigh here on the Earth's surface.]

Why can't you use $F = ma$ to find force (weight) 1000 km above the Earth's surface? Well, you could, if you knew the correct acceleration. The value for acceleration that I gave you, though, is at the Earth's surface. If you write

$$F = m \times a = \frac{Gm_1 m_2}{r^2}$$

you can see that

$$a = \frac{Gm_1}{r^2}.$$

In other words, acceleration due to gravity depends on the distance from the center of mass of the other body. So obviously acceleration due to gravity is going to depend on how far you are from the Earth's surface. Therefore, if you knew the gravitational acceleration 1000 km above the Earth's surface, you could use $F = ma$. But if you don't — and you probably don't — then you should use Equation 1 above (Equation 4 from class notes on 10 or 12 Feb).

My mass is the same on the moon (e), too: mass is always the same, everywhere, no matter what. So that's 73 kg again. So how much do I weigh on the moon? Again using the universal law of gravity (Equation 1 above), and now using the mass and radius of the moon instead of the Earth, (f) I get 118 N. You can see that this is around one sixth of what I weigh here on the surface of the Earth.

To find my weight on the moon in pounds, I would need to use the universal gravity law again, but there is a problem: I don't know G in English (American) units! One way to solve this is the following. I can calculate my weight on the moon in the same way as above. It turns out that I weigh about a sixth as much on the moon as I do on Earth. (Many of you looked up this "one sixth," but you were supposed to calculate these things.) My weight in pounds on the moon must also therefore be about one sixth of my weight on pounds on the Earth. (This is because gravitational acceleration depends on the mass of the bodies and the distance between the centers of mass. The masses and distances for me on the moon don't change no matter what units I use. Therefore the ratio must be the same. Come talk to me if this is not clear.) Therefore, if I weigh 160 pounds on the Earth, I must weigh (g) around 27 pounds on the moon.

Now we need my weight on Mars, in pounds and Newtons. My mass, of course, is the same on Mars as it is here: 73 kg, or 5 slugs. Using the Universal Law of Gravitation, I calculate that my weight on Mars (h) is around 252 N, which is about 35% of my weight here on Earth. There are now several ways to find my weight on Mars in pounds, but the easiest is this: if my weight on Mars in Newtons is 35% that of my weight on Earth in Newtons, then my weight in pounds on Mars must also be about 35% that of my weight on Earth in pounds: 160×0.35 or 56 pounds (h).

Another way that some people found their weights in pounds on the Moon and Mars was simply by using the fixed ratio between Newtons and pounds: 4.46 N equals 1 pound (you can find this by taking the ratio of 713 N and 160 pounds, or by doing the same with your own weights). Thus, if my weight on the Moon is 118 N, then my weight in pounds should be – using the Newtons/pounds conversion – around 26.5 pounds: pretty close, and close enough.

Remember: You cannot use a "standard" conversion between pounds and kilograms because *in general* it doesn't hold. Any conversion you found between those two only holds on the surface of the Earth! Obviously, on the Moon, there will be a different relationship between pounds and kilograms than on the Earth. A pound is a unit of weight: a function of the gravitational acceleration, that is, the mass and distance between the centers of mass of the attracting bodies. A kilogram is a unit of mass: bodies have the same mass everywhere.

2) The distance from the Earth to the Moon is around 400,000 km; the distance from the Earth to the Sun is around 1.5×10^8 km. The mass of the Moon is around 7×10^{22} kg, and the mass of the Sun is 2×10^{30} kg. So plugging in to the formula, you find that m^2/r^6 for the Moon is around 1.2×10^{12} (the units are kind of nonsense, kg^2/km^6. It doesn't really matter what the units are here, since we are going to compare them to each other, as long as they have the same units – kg or g but not both, m or km but not both.). For the Sun, the value is around 3.5×10^{11} (same units). So tides from the Moon are around 3 times stronger than those raised by the Sun – not all that much stronger! Note: you can win bets at parties with this information! This is why neap tide (when the Sun and Moon are at right angles with respect to the Earth) is a much smaller tide than spring tide – because the Earth is getting pulled in two different directions by forces that are not all that different from each other.

Many of you simply looked up the answer and did no calculation, and you got little credit for this. Others of you did the calculation – correctly! – but still parroted a slightly different version of text that was obviously copied from a web page. Have some confidence in your answers!

3) Seasons on Uranus are quite extreme. For example, the north pole of Uranus goes something like 20 Earth years without seeing the Sun – that's a cold winter. This occurs during the northern winter, when the Sun is shining full onto the south pole. This situation is reversed 40 Earth years (one half a Uranian year) later. Note that this is different than north and south pole summer and winter on the Earth: on the Earth, the poles never receive the most direct sunlight on the planet. On Uranus, at the solstices, the poles are receiving the most direct light from the Sun. Check out the rough figure I drew (Figure 1) – you could also have drawn a figure.

Also recall how I figured out how long a Uranian year is: using Kepler's third law, I said $P^2 = a^3$ where a is 19 AU. I can do this because a is in AU, P is in years, and the body is orbiting the Sun. I get a period of around 83 years.

Spring and fall are moderate, with every part of the planet receiving approximately equal Sun at the equinoxes.

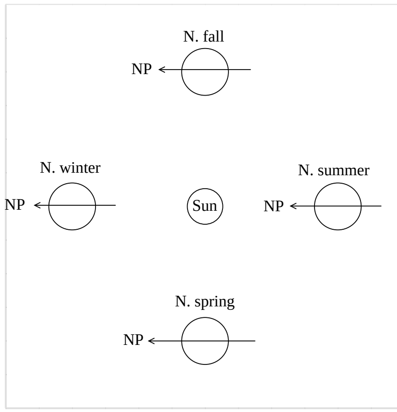

Figure 1: Seasons on Uranus. I have marked the Sun, Uranus, and Uranus' north pole. During northern winter, the north pole points directly away from the Sun and is never illuminated. The south pole points directly at the Sun and is essentially the hottest place on the planet. On Earth, the South Pole is never the hottest place on the planet. The lines across the planet show the rotation axis of the planet, with the arrow pointing at the North Pole.

Seasons last around 20 years each.

4) There are a bunch of different steps to this problem, and also several different ways to approach it. This is the way that I did it, but you might have done it a different way and still have done it correctly.

We know that $W = F \times d$. Here, F is the force that is being applied over some distance; in this case, this force is simply my weight, because that is what I am moving over a distance. There are a bunch of different ways to figure out your weight in Newtons (see above); I found that my weight is 715 N (your mileage may vary). Lastly, the distance we're talking about here corresponds to climbing three flights of stairs – it's only the vertical distance that matters. I'm going to very roughly guess 10 meters, although I know that's a bit high; in any case, it makes the math easier and doesn't really affect the answer. Thus, the work I am talking about is 7150 J.

You also know that 1 calorie (really kilocalorie) is equal to 4.184 kJ or 4184 J. Since 1 kilocalorie is 4184 J then the work done corresponds to 1.7 kilocalories, or 1.7 US RDA calories. This is about 0.1% of my US RDA caloric intake (around 2 out of 2000 RDA calories).

One mole of glucose is 180 grams: six carbons at atomic weight 12 each, for 72; twelve hydrogens at atomic weight 1 each, for 12; and six oxygens at atomic weight 16 each, for 96. To calculate the mass of a mole, you simply total the atomic weights, and give that number in grams: 180 grams. Metabolizing one mole of glucose releases 2870 kJ or 2,870,000 J. So if I perform 7150 J of work to climb the stairs then I need to consume 0.25% of a mole of glucose, or around half a gram. That's not so much! I guess I need to climb the stairs a lot every day (which I do) in order to call it sufficient exercise.

A single Reese's Peanut Butter Cup[1] is 21 g. This implies that 1/40th of a RPBC could power the climb to my office, or that I would need to climb the stairs 40 times to burn off one RPBC! Yikes! Further, you can

[1] See: `http://www.hersheys.com/reeses/nutrition-detail.asp?item=cups`

calculate that 1 g of glucose is about 3.8 (kilo)calories, whereas 1 g of RPBC is around 6.2 (kilo)calories (a single RPBC is 130 (kilo)calories for its 21 g.). Are RPBCs more energy packed than pure glucose? Should marathon runners chomp RPBCs before or during races?

This seems almost impossible – we would all weigh 1000 pounds if this were true. Well, what assumptions did we make? First, we assumed that your entire daily caloric intake was 100% glucose. It won't surprise you to learn that there are lots of other things in RPBCs besides glucose. In particular, there is a lot of fat – 7.5 g per RPBC, which, amazingly, is 10% (!!) of your RDA based on a 2000 calorie diet – in RPBCs. Fat has more calories per gram than basically any other food substance, which means it is a great energy storage medium (think about bears fattening up in order to prepare for their winter hibernation). So the fat in RPBCs carries a lot of calories. However, fat calories are much harder for the body to turn into energy than pure glucose, so the immediate energy boost from fat (and therefore RPBCs) is much lower than you might expect. In other words, we don't turn the 130 (kilo)calories in a RPBC directly into usable energy. Therefore, eating lots of RPBCs will give you a sugar high and an energy boost, but it will also give your body lots more fat than it actually needs. People with strict RPBC-only diets – like my college roommate who wrote the periodic table mnemonics – might find themselves fatter than they would like. (The roommate in question also had an amazing metabolism so that he would burn this fat faster than a normal human. Thus, a skinny guy.)

Bottom line: I really like RPBCs, but I try not to eat too many. People who need a lot of energy quickly will be best off by eating high-glucose, low-fat foods (think sugar candy). Marathon runners and long distance bikers tend to want not pure sugars, which your body easily digests and then is done with, but rather carbohydrates (think pasta) that take some time to be digested and turned into sugar by your body. Thus, there is a more constant supply of glucose to the exercising athlete over a longer time than the quick sugar rush from sugar candy.