Solutions: Homework set #2 (assigned 9 February, due 23 February)

General comments. Overall, I was pretty pleased with the work on this assignment. Many people did quite well. As always, please check these solutions carefully to see what you might have done wrong. Then feel free to come talk to me about your work.

1) You all certainly know from lectures and the readings that there are four RNA nucleic acid bases: A, G, U, and C. For any given polymer, each slot of the polymer can have any of these four bases in it. For a 4-monomer RNA strand, you have four choices of bases for the first slot, for the second slot, for the third slot, and for the fourth slot. Therefore, you have $4 \times 4 \times 4 \times 4$ possible four-monomer RNA strands: 256 possibilities (AAAA, AAAG, AAGA, AGAA, GAAA, AAAU, AAUA, AUAA, etc. – you can write them all down if you don't believe me!). Similarly, for a polymer that is 8 monomers long, you have $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$ possibilities, or 4^8, which is 65536 different unique potential 8-monomer RNA strands. To generalize, you might say that the number of combinations we have is equal to (number of bases)$^{\text{number of monomers}}$. Thus, if the RNA strand is 80 monomers long we have 4^{80} combinations: around 10^{48}. Recall that this is 1 followed by 48 zeroes. You can store a lot of information in genes! There are hundreds to thousands of bases per gene. So I can store 4^{100} ($\sim 10^{60}$) or perhaps as many as 4^{1000} (very big number, too big for my calculator) different combinations on one gene! It doesn't matter how many physical characteristics you think a human being has – it's not as many as the possibilities on one gene. Thus there is a lot of redundancy built into genetic information.

The protein problem is essentially the same problem but with different parameters. In a protein polymer that was made of four monomers, you could make 20^4 (160,000) different proteins. For an eight-monomer protein, there are 20^8 (2.56×10^{10}) different combinations. With an 80-monomer protein there are 20^{80} possible combinations – that's a whole lot. So even if your protein polymer is not made of as many monomer units as an RNA strand is, you still have an enormous number of possibilities for making proteins.

2) This question requires you to use Kepler's third law. Remember, in order to use the form $P^2 = a^3$, three things must be true: (1) P must be in years; (2) a must be in AUs; and (3) the mass of the body being orbited must be equal to the mass of the Sun. For this problem, #3 is not true, so you have to use the full-on form of the equation, as follows.

A geostationary orbit is one in which the satellite is always above the same spot on the Earth. Therefore, the period of the orbit is 24 hours, the same as the rotational period of the Earth. Since we know this and we know the mass of the Earth (6×10^{24} kg), we can use Kepler's third law to find the orbital distance (a). So we say

$$P^2 = \frac{4\pi^2}{GM} a^3 \tag{1}$$

and substitute in. You need to do this in seconds, meters, and kilograms in order to get the units to come out correctly. We plug in for P and M and find $a = 4.2 \times 10^7$ meters, or around 42,000 km. This is the distance from the Earth's center of mass, so the height above the Earth's surface is 42,000 km minus 6400 km, or around 36,000 km.

Low Earth orbit has an orbital period of around 90 minutes. We can use the same method to find what the orbital distance is. Plugging into Kepler's third law, we find $a = 6.7 \times 10^6$ meters, or around 6700 km above the Earth's center of mass. Since the Earth's radius is around 6400 km, this means that LEO orbits are about 300 km above the surface of the Earth.

A common mistake here was forgetting to subtract out the radius of the Earth.

3) As most of you are aware, it is hard to find and identify Archaea because they tend to live in weird places – places we might not ordinarily think of looking. Additionally, some of the places Archaea exist are hard

to look in (like the intestines of tube worms who live at ocean vents). Where would you look? Any answer that makes sense is acceptable here. A few I thought of include these: undersea volcanoes (heat source, water), deep underground (heat source, perhaps some water), Antarctica (very cold, but some organisms can survive this; salty; little water!), etc. You should give a complete and well-written answer; the specifics of your answer will depend on what you think. Your mileage may vary.

4) The traffic light problem was not very hard for most of you. If you missed many of these answers, you may need to review the material. My answers looked something like this.

	red	yellow	green
Wavelength	longest		shortest
Energy	least		most
Frequency	smallest		highest
Peak wavelength	longest		shortest