Solutions: Homework set #1 (assigned 26 January, due 4 February)

General comments. The homeworks were a bit all over the map – some excellent parts, some poor parts, but overall not too bad. There are a few points I want to emphasize: (1) In some places I wrote "causality" on your papers. This is because I believe you got the logical flow wrong, backwards, or circular. Please read carefully through these solutions to see why I might have marked that. Then come see me if you don't understand. (2) If I ever ask you again to write about some article that you choose, please please *please* make sure that you give a complete citation for that article so that I can find it. (3) Always, always make sure you answer the *why* part of the question. This is the most important part, and I always look for it.

Two more points: You need to always show your work. It is never sufficient to just write down an answer. And: **No brain dump:** Don't tell me irrelevant stuff in an attempt to beef up your answer.

Finally, although I don't explicitly take points off for spelling and grammar and punctuation, for some of you these details need a lot more attention. Eventually, laziness in this area will come back to bite you.

1) I am six feet tall. You can look up the fact that there are 39.37 inches in a meter. Therefore, I can convert units like this:

$$\text{(six feet)} \times \left(\frac{\text{12 inches}}{\text{1 foot}}\right) \times \left(\frac{\text{1 meter}}{\text{39.37 inches}}\right) = 1.83 \text{ m}$$

where all the quantities in parentheses are ratios that are equal to 1. These are the conversion factors.

I grew up in Durham, NC, which is about 2000 miles from Flagstaff. 1 mile is around 1.6 km, so, using the technique above, I find that Durham is 2000 miles × (1.6 km/1 mile) = 3200 km from Flagstaff.

I think that my car is about 15 feet long, about 5 feet tall, and about 6 feet wide (close enough). That means that the volume is 15 feet × 5 feet × 6 feet, which is 450 ft^3 (did you get the units correct?). Now, watch this:

$$450 \text{ ft}^3 \times \left(\frac{1 \text{ m}}{3.28 \text{ ft}}\right)^3 = 12.8 \text{ m}^3.$$

It is important that you remember the cube both the top and the bottom of that conversion factor – that way the units cancel out correctly. *Many people messed this up – make sure you understand it.*

Now the rest should be easy. A basketball court is 94 feet by 50 feet, which is 4700 ft^2. Using 1 m equals 3.28 ft, I find that the area is 4700 ft^2 × (1 m/3.28 $\text{ft})^2$ which is 437 m^2. Remember that there are 100 cm in 1 m, so there must be 437 m^2 × (100 cm/1 $\text{m})^2$ which is 4.37×10^6 cm^2. (This might be a good time to remember scientific notation and how to make your calculator do it – come see me if you need a refresher.)

The NAU Skydome (not the one in Toronto!) has a radius of 251 ft and a height of 142 ft – assume it is a cylinder and forget about the domed up part. I told you the formula for volume of a cylinder was $\pi r^2 h$, so you should have been able to find that the volume was 2.8×10^7 ft^3. Make sure you can find that this volume is equal to 7.8×10^5 m^3, which is also equal to 7.8×10^{-4} km^3 (1000 m in 1 km) and also equal to 7.8×10^{11} cm^3. Many people didn't get this correct – don't forget to cube the entire conversion factor.

These are very important skills, both for this class and for life. You will need to do all of these units and conversions tasks again in this class, so make sure you can do it correctly!

2) We have seen in lecture several times a figure that shows the relative proportions of elements in the presolar nebula — the primordial cosmochemical abundance of elements at the beginning of the Solar System. It should be pretty straightforward, then, for you to identify the fact that the Earth is, first and foremost, dramatically depleted in H and He compared the presolar nebular cosmic abundances.

Clearly, the Earth is quite depleted in hydrogen and helium compared to the pre-Solar System cosmic abundance. Generally, this is because when the Earth was forming as a ball of rocky stuff in the gaseous protoplanetary nebula, it [the Earth] was not massive enough to retain most of the nebula's H and He gas. Thus, when the H and He dissipated from the protoplanetary nebula, the Earth was left behind, depleted in these elements.

One of the Earth's most abundant elements is oxygen; oxygen is also the next most common element in the presolar nebula after H and He. However, in the bulk Earth, oxygen is followed by magnesium and silicon, but the presolar nebula finds Mg and Si way down on the list, around 30 times less abundant than oxygen (compared to half as abundant on the bulk Earth). Mg and Si are therefore relatively enhanced on the bulk Earth. Similarly, the most abundant element in the bulk Earth is iron (Fe), whereas iron is also about 30 times less abundant than O in the presolar nebula.

In other words, the lightest gases in the presolar nebula did not get trapped and retained in/on the Earth, so the Earth is relatively enhanced in heavier elements and relatively depleted in H and He.

Jupiter, on the other hand, has an abundance that is very similar to the presolar cosmic abundance. This is because Jupiter *was* massive enough to hold onto H and He from the protoplanetary nebula when the planets were forming. Thus, its composition is very similar to the cosmic abundance.

The Earth's lithosphere is somewhat similar to the bulk Earth, but with more carbon, nitrogen, oxygen, and silicon and significantly less magnesium and iron. This is because the rocks that make up the lithosphere are predominantly made of things like silicon (quartz). The core of the Earth, on the other hand, is largely iron. The Earth is *differentiated*, with the denser (heavier) elements in the core and the lighter elements riding on top (in the lithosphere). This is simply a result of gravity – the heavier elements gradually sank to the core during the formation of the Earth.

The Earth's biosphere (by which I mean "all living things on Earth") is even more enhanced in carbon and oxygen and somewhat depleted in nitrogen compared to the lithosphere. Naturally, this is because most living things are made of carbon and oxygen.

One implication that all this has for the search for life in the Universe is that a relative enhancement of carbon (but not necessarily oxygen!) could potentially be and be used as a biomarker (a signature of life). While this would not ensure life (of course), it may be that such an enhanced concentration could suggest an increased likelihood of life there. Indeed, it is probably the relative enhancement of carbon in the Earth's lithosphere that first enabled life to originate on our planet!

There were a few repeated errors. One is the causality issue. For example, life does not affect the bulk composition of the Earth or the lithosphere (life is a very, very small component), but the composition of the lithosphere could affect life. Similarly, the oxygen in the bulk Earth long predates life on Earth, so you can't use life to explain the large amount of oxygen in the bulk Earth. Another issue is that the composition of the Earth has not changed essentially since it formed 4.5 billion years ago (with the exception of minor radioactive decay). Neither the Earth nor Jupiter is fusing elements right now, and the Sun's fusion of H to He is basically irrelevant. Also, Jupiter and the Earth formed out of the same solar nebula at the same time. Several of you suggested that the Sun's gravity was important, which is half right – gravity matters – but also half wrong – it's not the Sun's gravity that matters, but Earth's and Jupiter's.

More general comments: It is circular reasoning, and not very interesting, to say that Jupiter is made of gas because it is a gas giant planet. Also, if I ask you to compare things, I don't want you to just give me a run down of the numbers, because I assume you can read. Instead, I want you to point out the interesting ways in which the two things are similar (or different).

3) CH_4 is a very good example of a covalent bond. Both the C and the Hs are happy because their outer electron levels are filled with 8 and 2 electons each, respectively, through covalent electron sharing. Same goes for NH_3 because nitrogen has five electrons in its outer shell and would like to have three more that it shares covalently with the hydrogens. Carbon dioxide has a nice double bond between the oxygens and the

carbon – two electrons are shared between each oxygen and the carbon so that each of the three elements has a filled outer shell (carbon has four and shares four more with the two oxygens). It turns out that CO_2's bond is somewhere in between ionic and covalent, so any answer you wrote that makes sense was accepted. All these molecules were common in the presolar nebula because they all have bond situations in which the elements involved have filled electron shells, which is the preferred state for elements.

NH_4 on the other hand does not exactly fill the outer electron level of the nitrogen or of one of the hydrogens. If the nitrogen and three of the hydrogens covelently share their electrons, then you still have one hydrogen left over that has a single electron and an unfilled electron level. This is not a preferred state; hence, this molecule is not as common. SiO_2 looks pretty good, actually, with a nice double bond just like CO_2. Like CO_2, this bond is somewhere in between an ionic and covalent bond (closer to ionic than CO_2's bonds – ask me if you want details). So what's the problem – why was this molecule relatively rare in the presolar nebula? Because — as you know well by now — silicon was fairly rare in the presolar nebula, so there just wasn't much around to bond to the oxygens. Also, a lot of the oxygen was already bound up with CO_2, H_2O, and other common molecules, so SiO_2 was out of luck. However, in an environment in which silicon is relatively very abundant – like the Earth's lithosphere, for example – SiO_2 is pretty common since that is a favorable bonding situation. SiO_2 is found in sand, sandstone, granite, etc. and is quite common in the lithosphere.

You can probably see where we are going with all this. NaCl represents a nice ionic bond, but neither of those elements was particularly common at the time of the Solar System's formation, so NaCl was not a particularly common molecule then. CO is made of common elements, but a double bond between the C and the O still leaves C with an unfilled outer electron shell – thus this is not a preferred molecule and would have been less common in the early Solar System.

There was some amount of confusion on this problem. Please come talk to me if you do not understand the answer I have written. Note that it is not really true that covalent bonds must be stronger than ionic bonds. They can be, but don't have to be.

4) There are a lot of steps to this problem, but if you lay them all out one at a time, you should have no trouble doing them. Remember that keeping track of the proper units is a good way to make sure that what you are doing makes sense.

You are asked how much heat (energy) was created in the first 1.5 million years of the Earth's existence through radioactive decay of ^{26}Al. You know how much energy the decay of 1 ^{26}Al atom releases: 2.9×10^{-13} J. So all you have to do is figure out how many ^{26}Al atoms decay over the first 1.5 million years.

The mass of the Earth is 6×10^{27} grams. We know, from Table 1, that today aluminum makes up around 1.59% (by mass) of the Earth. (Note: this has changed only an imperceptible amount due to the decay of ^{26}Al.) Today there is very little ^{26}Al in the Earth (it has all decayed), but we know that when aluminum is created, the fraction of all aluminum that is ^{26}Al is 5×10^{-5}, and we assume that this aluminum created in stars is immediately incorporated into the Earth. The mass of ^{26}Al in the Earth when the Earth formed (assuming no decay before the Earth reached its final mass) is therefore

$$(6 \times 10^{27}\ \text{g}) \times 0.0159 \times (5 \times 10^{-5}) = 5 \times 10^{21}\ \text{g of } ^{26}\text{Al}.$$

Note that the amount of Al on the Earth today is basically the same as it was when the Earth formed: ^{26}Al is important for this problem, but negligible for the total amount of Al on Earth. We also know that one gram of ^{26}Al has 2×10^{26} atoms in it. (Actually, I messed this up – I had the wrong number for a mole in the problem set – sorry! But I'll continue this solution with the wrong number so you can see how you should have done it.) Therefore, the total number of ^{26}Al atoms in the Earth (before decay) was

$$(5 \times 10^{21}\ \text{g}) \times (2 \times 10^{26}\ \text{atoms/g}) = 10^{48}\ ^{26}\text{Al atoms}.$$

The only remaining question, then, is how many of these initial ^{26}Al atoms decay over 1.5 million years. The half-life of ^{26}Al is 720,000 years, so 1.5 million years is about two half-lives. Since half of the ^{26}Al decays in the first 720,000 years, and half of the remaining ^{26}Al decays in the next 720,000 years, about three quarters of the initial amount has decayed after two half-lives. Thus, the number of ^{26}Al decays over 1.5 million years is around 7.5×10^{47}.

Now we are on the home stretch. Each decay releases 2.9×10^{-13} J, so the total amount of energy released is

$$(7.5 \times 10^{43} \text{ atoms}) \times (2.9 \times 10^{-13} \text{ J/atom}) = 2.2 \times 10^{35} \text{ J}.$$

We can convert this to power to facilitate comparisons. Power is the amount of energy (Joules) emitted in a certain amount of time (in this case, 1.5 million years). Thus,

$$\text{Power (W)} = \frac{2.2 \times 10^{31} \text{ J}}{1.5 \times 10^{6} \text{ years}} \times \left(\frac{1 \text{ year}}{365 \text{ days}}\right) \times \left(\frac{1 \text{ day}}{24 \text{ hours}}\right) \times \left(\frac{1 \text{ hour}}{60 \text{ minutes}}\right) \times \left(\frac{1 \text{ minute}}{60 \text{ seconds}}\right).$$

The reason we have to convert everything to seconds is because Watts are defined in terms of Joules and seconds (1 W is 1 J per second). When we multiply all those numbers out, we get 4.7×10^{21} W. Thus, the decay of ^{26}Al in the early Earth could have powered 8×10^{19} lightbulbs, 60 W each, for 1.5 million years! This is 200 million billion lightbulbs. However, this is but a tiny fraction of the Sun's power output: $(4.7 \times 10^{21} \text{ W})/(3 \times 10^{26} \text{ W})$ is 2×10^{-5}. The Sun is 64,000 times more powerful than the decay of ^{26}Al in the early Earth.

If you didn't follow all my steps, or didn't make those steps on your own, you should look over what I've written here and make sure that it all makes sense. More importantly, you should start to see how to lay out complicated problems into simple pieces that can be solved in order, one at a time. Please do come and see me if you have questions about this.

For future assignments, *make sure that you always show all your work!*

5) Nearly any coherent answer was acceptable here. A common gotcha: You must provide the citation of the article you read so that I can find it, even for (especially for) web articles. Some of you need some more practice reading articles like this and writing about them, and we'll continue to work on that this semester. Also, a few of you chose articles that were not primary news articles. You should learn to tell the difference between real journalism and press releases. Also, a few people chose articles that were interesting, but not related to astrobiology.