PHY 481/581 - Quiz 5

Name: Solutions

Date: 11/19/2018

Electronic band theory

Problem 1. Describe Bloch's theorem both qualitatively and quantitatively. For the latter, write down at least one mathematical equation that is the theorem. Next describe the significance of the interpretation of this theorem when comparing the behavior of a Bloch wave to the classical Drude theory.

Problem 2. Draw a two-dimensional version of the Fermi surface for the free electron for a valence-one metal. Hint: the first Brillouin zone is exactly half full. Next draw the Fermi surface for the same metal in a periodic potential (again in 2D). In a sentence or two, describe the difference between the shapes of the two Fermi surfaces.

Problem 3. For a one-dimensional metal, draw the dispersion curve for the free electron model. Next, do the same for the nearly free electron model. Briefly describe the physical reason for the difference between the two curves.

Problem 4. Draw representative one-dimensional dispersion curves for an electrical insulator, conductor, and semiconductor identifying the regions of allowed and disallowed energies.

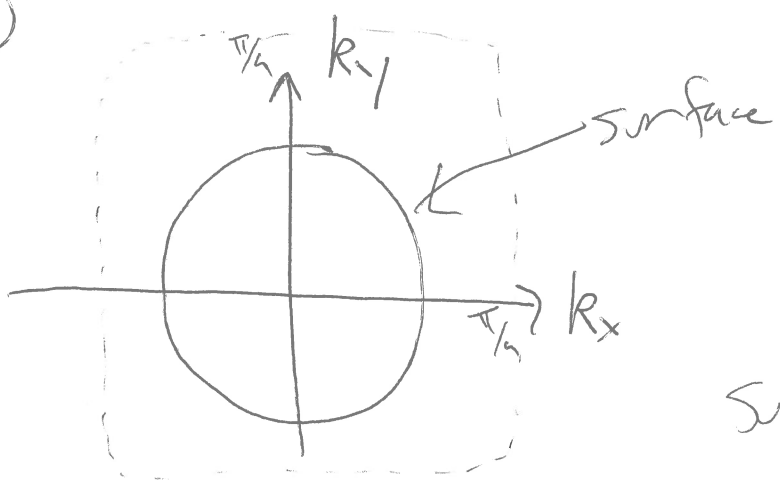
(1) the eigenstates of a wave travelling in a periodic Bravais lattice can be chosen to have the form

$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}\, u(\vec{r})$$

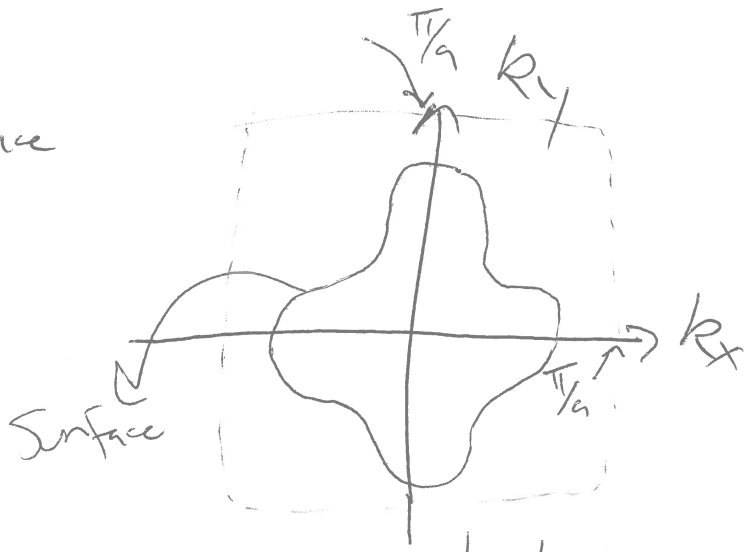
where $u(\vec{r}) = u(\vec{r}+\vec{R})$ has the same periodicity as the lattice

- alternate form: $\psi(\vec{r}+\vec{R}) = e^{i\vec{k}\cdot\vec{R}}\,\psi(\vec{r})$
- the primary significance is that the speed does not decay in time, as in the Drude model, & electrons may pass through the lattice virtually unimpeded.

(2)



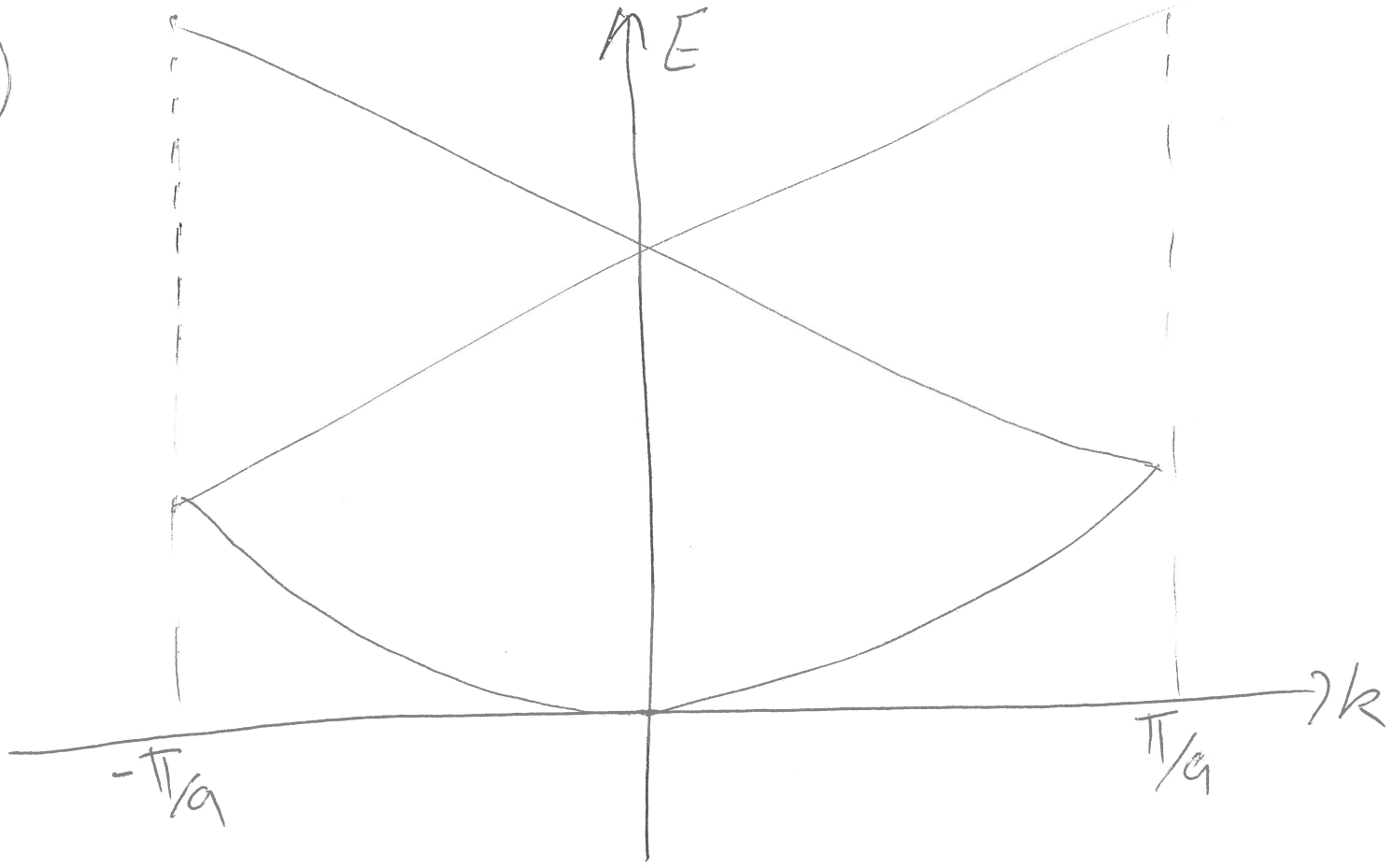
Free electron

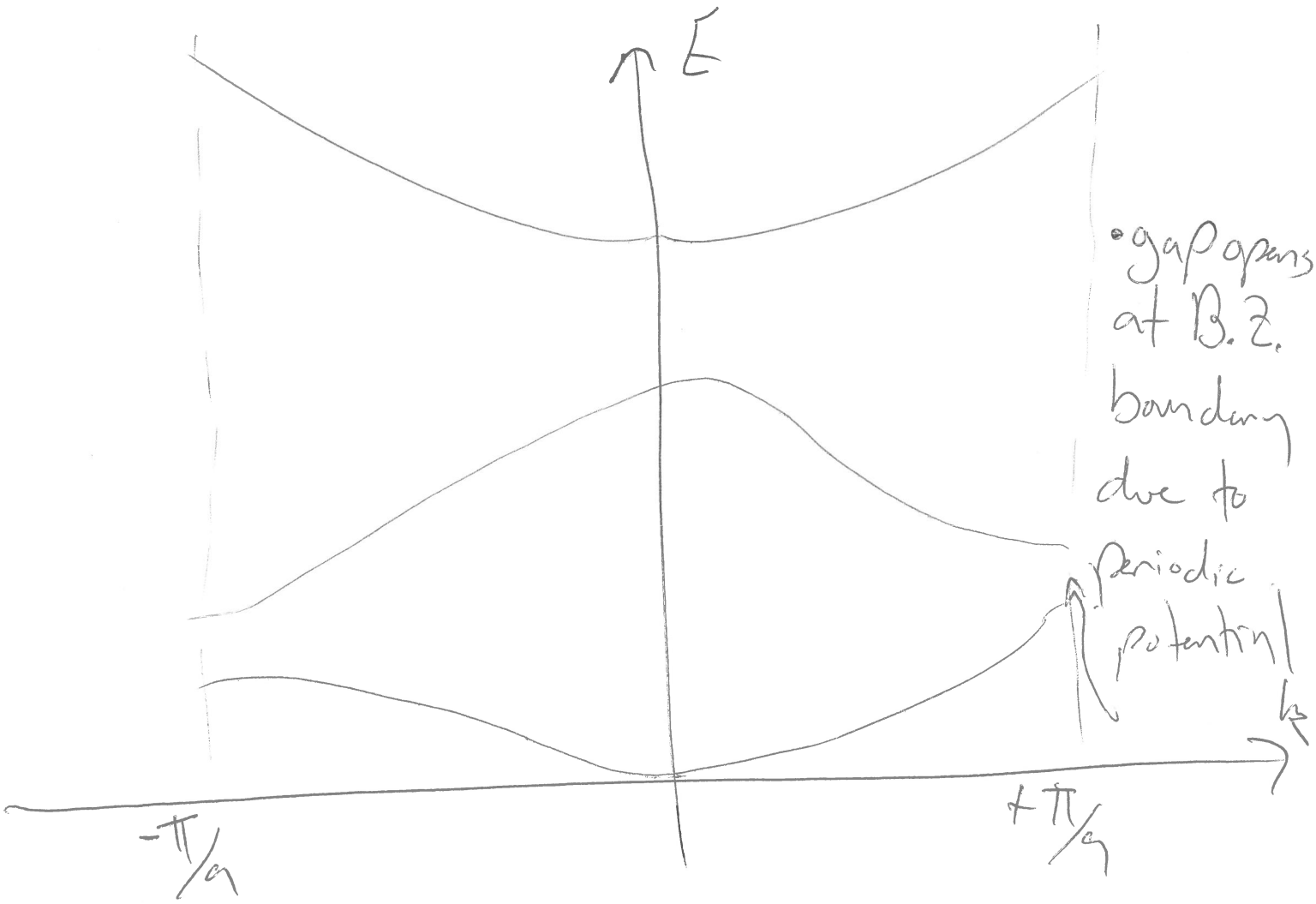


periodic potential

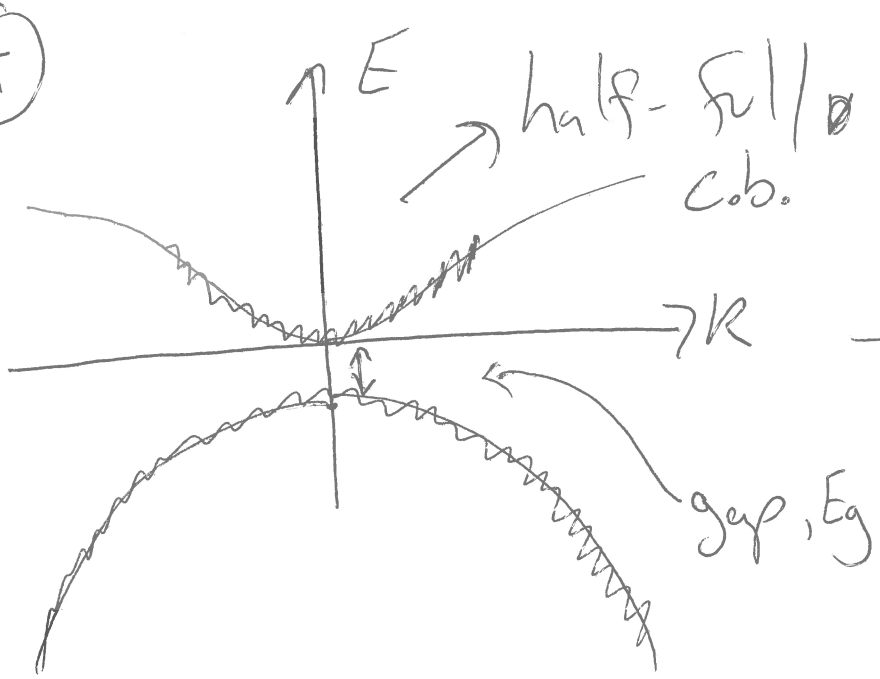
- the states closest to the zone boundary are "pushed" down in energy the most
- thus, those states are filled preferentially at the expense of states further from boundary

(3)

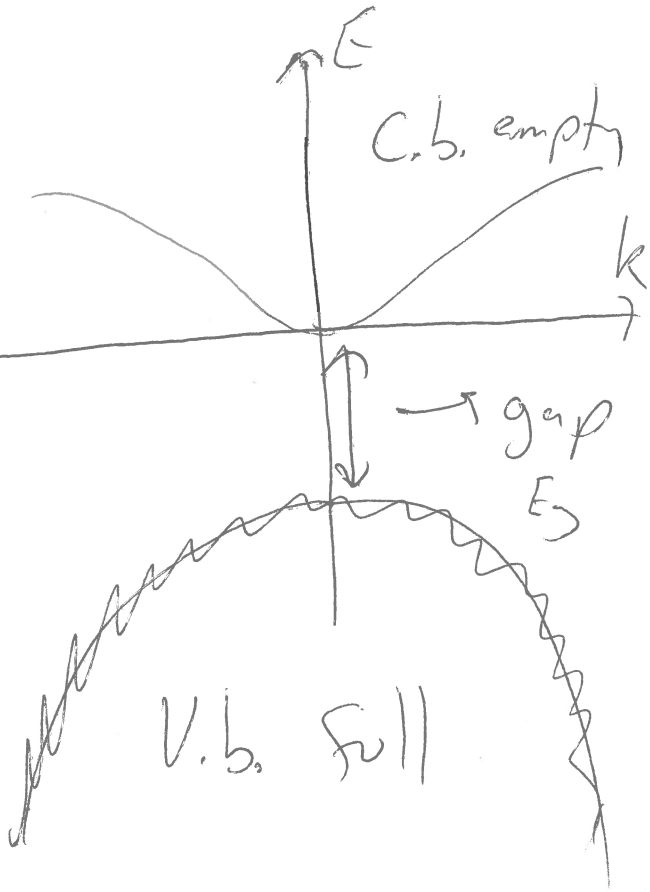

E
k
$-\pi/a$
π/a

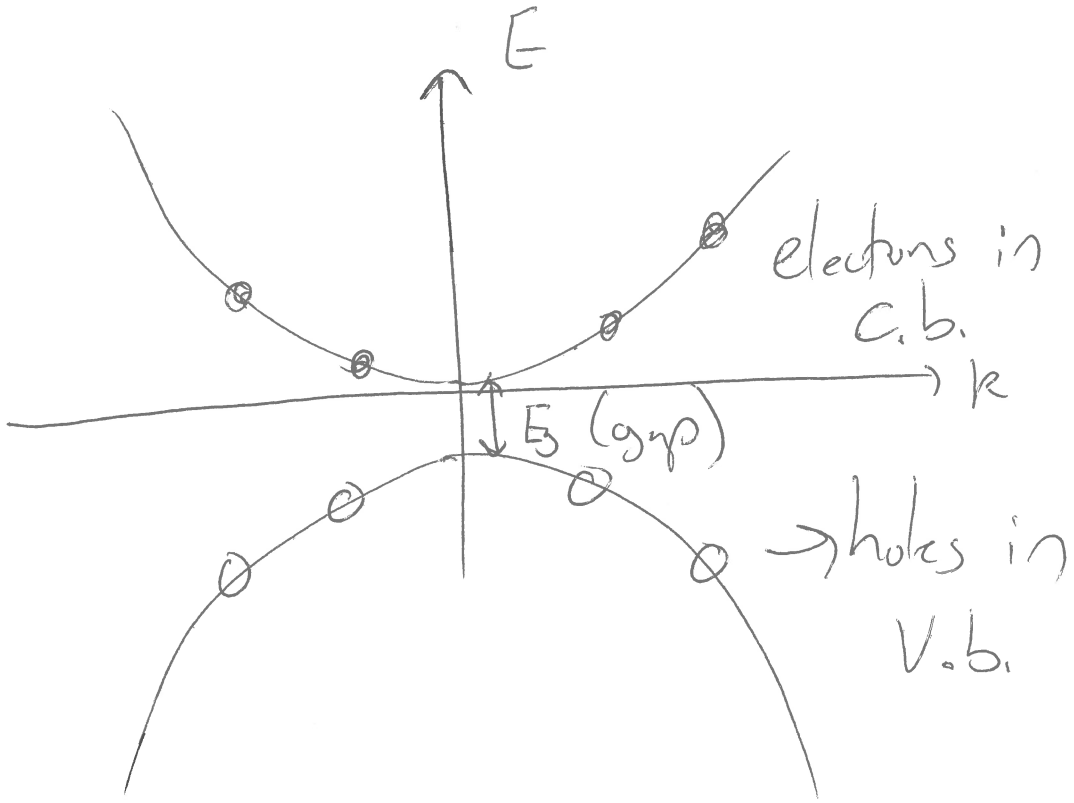

E
k
$-\pi/a$
$+\pi/a$
• gap opens at B.Z. boundary due to periodic potential

(4)


E
half-full
c.b.
K
gap, Eg

Metal


E
C.b. empty
k
gap
Eg
V.b. full


E
electrons in
C.b.
k
Eg (gap)
holes in
V.b.