PHY 481/581 - QUIZ 3

Name: Solutions

Date: 10/15/2018

1 Elementary Models of Vibrations in Solids

Problem 1. For a continuous medium, the dispersion relation is given by, $\omega = vk$. However, we found that the 1D lattice, consisting of equally spaced masses (of equal mass, m) and equal spring constant, α (Figure 1), shows a different dispersion relation, $\omega = \omega_{max}|\sin(ka/2)|$, where ω_{max} is the maximum possible frequency.

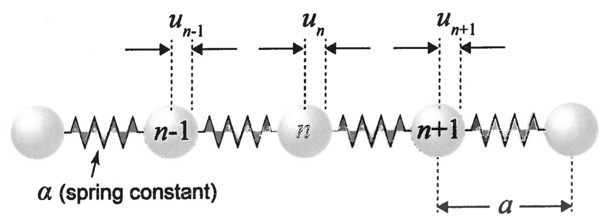

Figure 1: Model of the 1D monatomic solid. u_n is the displacement of atom n away from equilibrium.

(a.) Show that the dispersion relation for the discrete masses approximates the dispersion relation for the continuous medium for long wavelength, $\lambda = 2\pi/k$.

(b.) If the equilibrium spacing of the masses in the 1D monatomic solid is a, determine the value of ω at the first Brillouin zone boundary. Determine the group velocity at the boundary, and briefly describe the physical reason for this result.

(c.) Using the schematic in Figure 1, write down the equation of motion for atom n, assuming it is only coupled to its two nearest neighbors. Note: in Figure 1, u_n is the displacement of atom n away from its equilibrium position, $x_n = na$. Now write down your "Ansatz," (expected solution to your diff. eq.).

Problem 2. Assume a 1D *diatomic solid* has the same geometry as the monatomic lattice (Figure 1), but every other mass is now m_2, and each atom of mass m_2 has two nearest neighbor atoms of mass m_1.

(a.) Sketch the dispersion relation. Make sure to label the important features of your plot.

(b.) What is the significance of the frequency gap?

(c.) Describe the difference between this dispersion relation and that of the 1D monatomic solid.

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(a) $\lambda = 2\pi/k$

$\sin\left(\frac{ka}{2}\right) \simeq \frac{ka}{2}$, for $k \ll 1$

$\Rightarrow \omega \propto k$, linear

(b) boundary values: $k = \pm \pi/a$

$\omega = \omega_{max}\left|\sin\left(\frac{\pm\pi}{a}\cdot\frac{a}{2}\right)\right|$

$= \omega_{max}\left|\sin\left(\pm\pi/2\right)\right| = \omega_{max}$

$V_g = 0$ as $V_g \propto \cos\left(\pm\frac{\pi}{2}\right) = 0$

(c) $m\ddot{u}_n = -\alpha\left[2u_n - u_{n+1} - u_{n-1}\right]$

$u_n = \left(e^{i[kna - \omega t]}\right)A$

$x_n = na$ amplitude

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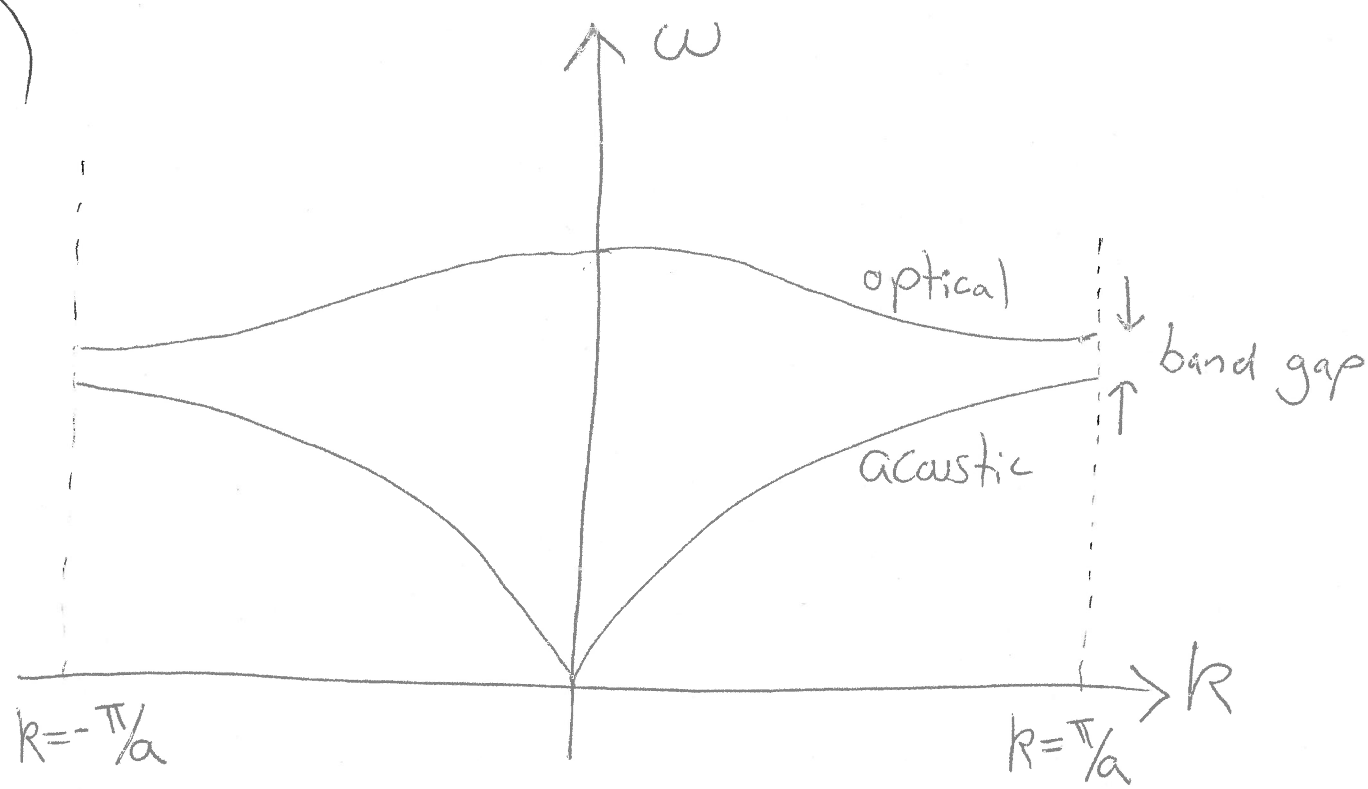
(a)



(b) the crystal cannot support modes at these frequencies

(c) for the acoustic branch, we have long-wavelength modes in which the atoms move together (~ in-phase); for the optical branch, the atoms move ~ 180° out-of-phase, or against one another.