# PHY 481/581 - Quiz 2

Name: Solutions  Date: 09/28/2018

**Problem 1.** What are the basic assumptions of the classical Drude free electron model? (hint: there are three)

**Problem 2.** Drude theory was fairly decent at modeling some effects in certain situations. An experimental apparatus for measuring the Hall effect is shown schematically in Fig. 1. Given a constant current, $I$, constant magnetic field, $B$, cross-sectional area through which $I$ flows, $Wt$, electron charge $-e$, and charge carrier density $n$, determine the *Hall voltage*, $V_H$, measured perpendicular to the current flow (in terms of the given parameters).

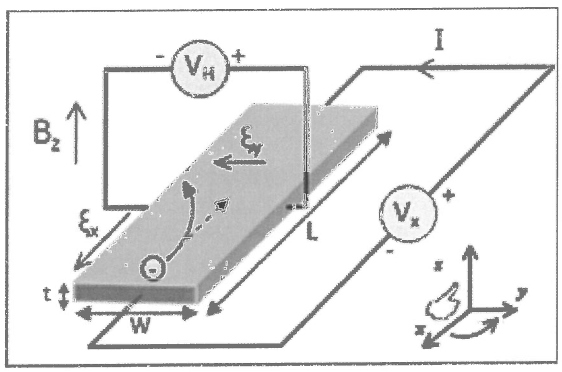



Figure 1: Schematic of an experimental apparatus to measure the Hall voltage $V_H$.

**Problem 3.** Explain why Drude theory is limited and how Sommerfeld's theory was more accurate.

**Problem 4.** Fermi-Dirac statistics is modeled by the following distribution

$$n_F = \frac{1}{e^{\beta(E-E_F)}+1}, \tag{1}$$

where $E_F$ is the *Fermi energy*. Sketch this function for $T=0$, and for $T \approx T_R$, where $T_R$ is room temperature. Make sure to clearly label your plot with axes, etc.

**Problem 5.** Explain the difference between the two plots in Problem 4, for $T=0$ and $T \approx T_R$. Then, approximate the low-$T$ dependence of the electron heat capacity. Hint: assume each electron, that *can* be excited, absorbs $\approx k_b T$ of heat.

**Problem 6.** Determine the number of *free* electron states, $N$, in a solid of volume, $V$. Hint: the states fill one octant of a sphere with a radius, $k_F$, in *k-space*. Using this expression for $N$, solve for the *Fermi energy*, $E_F = \frac{\hbar^2 k_F^2}{2m}$.

(1)

(a) electrons have a scattering time with probability $dt/\tau$ for time interval $dt$

(b) after a scattering event, the electron returns to $\vec{p} = 0$

(c) the electrons, being charged, respond to $\vec{E}$ + $\vec{B}$ fields

(2) $\vec{F} = -e(-\vec{E} + \vec{v} \times \vec{B}) = 0$

$$\Rightarrow \vec{E} = \vec{v} \times \vec{B} \Rightarrow E_y = v_x B_z$$

$E_y = \frac{-V_H}{W}$, $V_H \equiv$ Hall voltage

$V_H = -v_x B_z W$

↑ voltage ↑ velocity

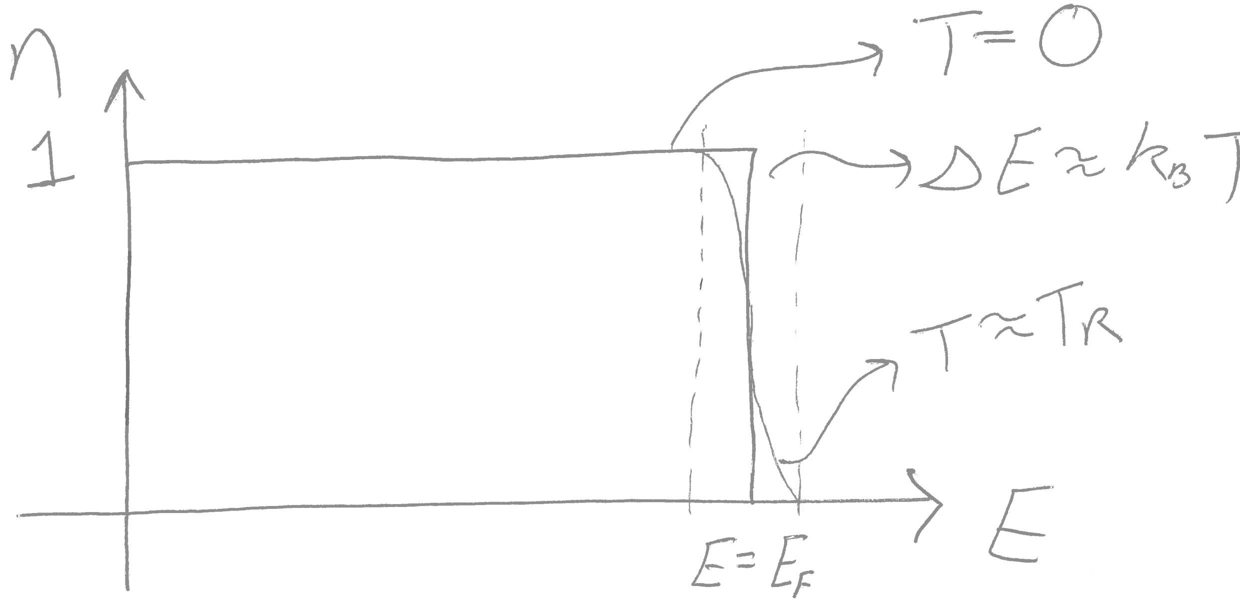
$I = (ne) \cdot \text{Area} \cdot v_x$, where $(ne) = \rho$, so $I = \rho \cdot v_x$

$I = -ne\,t\,W\,v_x$

$$W = \frac{-I}{net\,v_x} \Rightarrow V_H = -v_x B_z \left(\frac{-I}{net\,v_x}\right) = \boxed{\frac{B_z I}{net} = V_H}$$

(3) Drude theory does not take into consideration Fermi-Dirac statistics. That is, in Drude theory, the electrons can all be in the same "state," but according to F-D stats. this assumption is not correct. The Sommerfeld model <u>does</u> take into consideration F-D statistics by acknowledging the Pauli exclusion principle.

(4) $n_F = \left(e^{\beta(E-E_F)} + 1\right)^{-1}$




(5) @ $T=0$, all states have a 100% chance of being occupied below the Fermi energy. For $T \approx T_R$, some of the electrons near $E_F$ can be excited above $E_F$, but only a fraction $\simeq \frac{k_B T}{E_F}$, of all electrons, i.e.,

only electrons near $E_F$ are excitable.

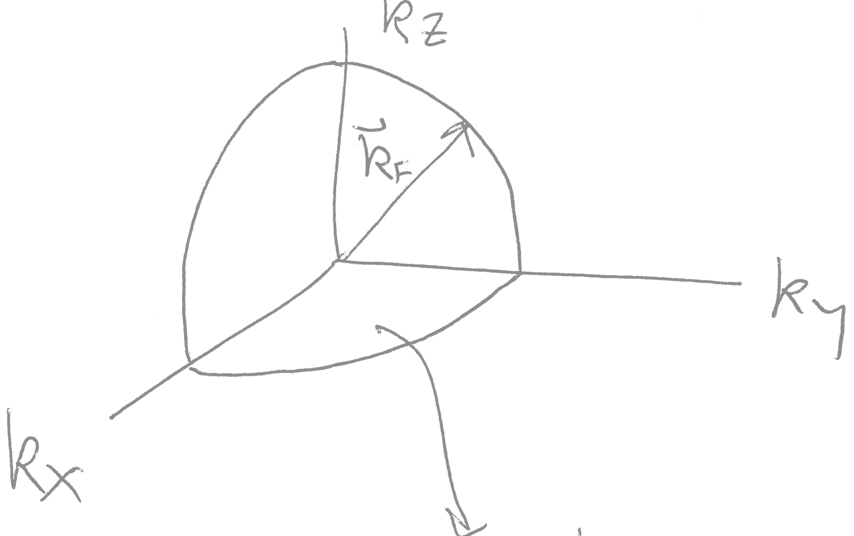
# #5 continued:

is $\sim \frac{k_B T}{E_F}$ electrons can be excited, only this many may absorb thermal energy. Thus, the total energy of excitable $e^-$'s is $\sim \left(\frac{k_B T}{E_F}\right) k_B T$

- the approximate heat capacity is $C_V = \frac{\partial E}{\partial T} \sim \frac{2 k_B^2 T}{E_F}$

$\therefore \; C_V \propto T$ (linear)

(6)




volume of this octant

$$\frac{1}{8}\left(\frac{4}{3}\pi k_F^3\right)$$

- For a 1-D solid, the number of states is $\frac{L}{\pi}$

→ for 3-D, we have $\left(\frac{L}{\pi}\right)^3$, for a cube of volume $= L^3$

- two $e^-$s ← $N = 2\left(\frac{L}{\pi}\right)^3 \frac{1}{8}\left(\frac{4}{3}\pi k_F^3\right)$

$$= \frac{V}{3\pi^2} k_F^3$$

#6 continued

$$N = \frac{V}{3\pi^2} k_F^3$$

$$\Rightarrow k_F = \left(\frac{N}{V} 3\pi^2\right)^{1/3}$$

$\frac{N}{V} = n$, density of free electrons

$$E_F = \frac{\hbar^2 (k_F)^2}{2m} = \boxed{\frac{\hbar^2 (n 3\pi^2)^{2/3}}{2m}}$$

$E_F \propto n$, density of free electrons