PHY 481/581 - Homework Set 2

Northern Arizona University Due: 10/01/2018

Solutions

1 The Free Electron Model

Problem 1. Define mathematically the Hall Coefficient as described by Drude theory. Estimate the magnitude of the Hall voltage for a specimen of sodium (Na) in the form of a rod of rectangular cross-section, with dimensions 5 mm $\times$ 5 mm. The rectangular rod carries a current of 1A down its axis in a magnetic field of 1T that is perpendicular to the direction of the current. The density of Na atoms is $\approx$ 1 gram/cm^3, and Na has an atomic mass of $\approx$ 23. Assume there is one free electron per Na atom, or Na has valence 1.

Problem 2. Sketch the distribution function of a Fermi gas for (a) temperature $T = 0$K and (b) for some temperature so that $k_B T \ll E_F$, where E_F is the "Fermi energy." That is, the thermal energy, $k_B T$, is a small fraction of the Fermi energy for the second sketch.

Problem 3. Explain why only a small fraction of the electrons can be thermally excited in a free electron gas that obeys Fermi statistics. Describe what effect this thermal excitation has on your sketches in Problem 2, and approximate the fraction of electrons that are expected to be excited as a function of thermal energy.

Problem 4. From Problem 3, if only this fraction of all free electrons, given by N, can be thermally excited, estimate the total thermal kinetic energy of the electrons, U_e. Recall that the thermal energy of a single particle of a classical gas is $\approx k_b T$.

Problem 5. From your estimation of the thermal energy of the electrons (electronic thermal energy), from Problem 4, take one further step to estimate the temperature-dependence of the electronic heat capacity. Keep in mind this is an approximation, not an exact solution.

Problem 6. Recall the low-T dependence of the heat capacity that arises from atomic vibrations (the Debye model, look it up). Combine Debye's result with your own result for electronic heat capacity (Problem 5) to obtain a more accurate low-T expression for heat capacity. In what temperature range does the electronic heat capacity dominate?

① Equilibrium occurs when $\sum F = 0$

$$\vec{F}_{mag} = -e(\vec{v}\times\vec{B}) = -evB\hat{y}$$

$$\vec{F}_E = e\vec{E} = eE_H\hat{y}$$

$$E_H = vB \text{ , Hall field}$$

$$\vec{J} = \rho\vec{v} \text{ ,}$$

$$= -en\vec{v} \text{ , } \rho = -en$$

$$vB = -\frac{JB}{ne}$$

$$E_H = \frac{-JB}{ne} = +R_H JB$$

$$\boxed{R_H = -\frac{1}{ne}}$$

#1 continued:

Na

Area = 5mm x 5mm

$I = 1A$

$B = 1T$

$$R_H = -\frac{1}{ne} = -\frac{1}{(1000 kg/m^3)(1.6 \times 10^{-19} C)}$$

$$V_H = E_H \cdot 5 \times 10^{-3} m$$

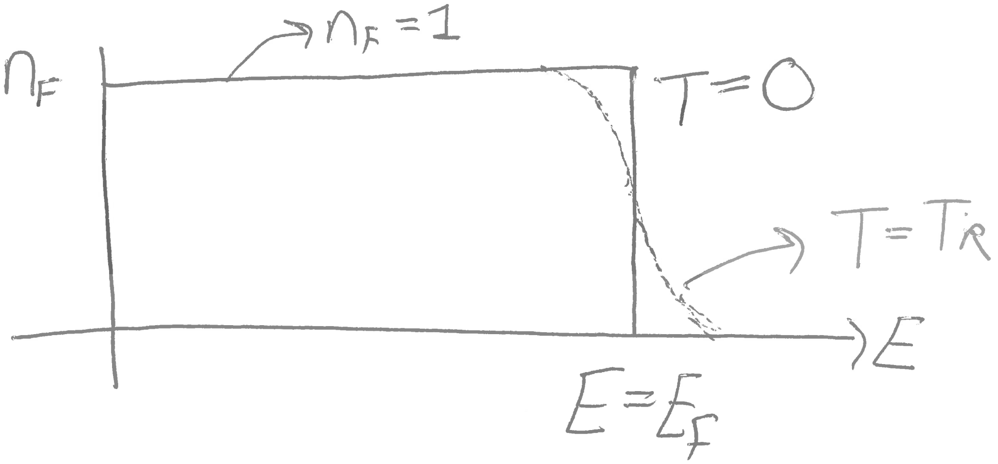
$$\uparrow$$

$$-\frac{JB}{ne} \qquad J = 1A / (5 \times 10^{-3} m)^2$$

$$V_H = \frac{(1A)(1T)(0.023 kg/mol)}{(1000 kg/m^3)(1.6 \times 10^{-19} C)(5 \times 10^{-3} m)(6.02 \times 10^{23} e/mol)}$$

$$\approx 5 \times 10^{-8} V$$

(2)



(3) only a small fraction $\sim \frac{k_B T}{E_F}$ can be excited because the electrons in the lower states cannot gain energy due to the other electrons of $\sim k_B T$ higher energy already being occupied

* as T increases, the most energetic electrons can be excited above E_F, thus fewer electrons are found just below E_F & the same # absent below E_F are present above E_F. Thus, higher T plots are "spread out."

(4) the spread in the plot is $\sim k_B T$,

thus $\sim \frac{k_B T}{E_F}$ electrons are excited

at some $T \neq 0$.

- If the number of excited electrons is $\sim \frac{k_B T}{E_F} N$ (fraction of total # N)

then we expect $\sim \left(\frac{k_B T}{E_F}\right) N \cdot k_B T$, since each electron can absorb $\sim k_B T$ energy.

- so, $E \approx \frac{(k_B T)^2 N}{E_F}$

(5) $\Rightarrow$ $C_V = \frac{\partial E}{\partial T} = \frac{2 k_B T N}{E_F} \propto T$

⑥ the Debye model predicts a T^3 - dependence on C_V, while for metals there is also a T- dependence at very low T.

$$C_V = \alpha T + \beta T^3$$

* The αT- term dominates at $T < 0$ & the βT^3 - term dominates at $T > 0$