7/42

Wed. Sept. 12

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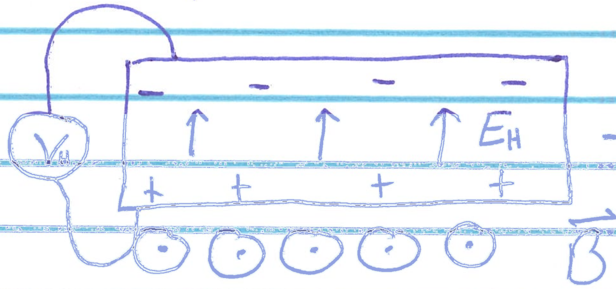
- last time we discussed the Hall effect
- our example was a "slab" of metal



- apply $\vec{J}$ + $\perp$ $\vec{B}$

- measure V_H, the "Hall" voltage and convert to E_H
- we found $E_H = \left(\frac{-1}{ne}\right) JB = R_H JB$

  Hall coeff. R_H

- We supply J + B + make measurement E_H
- $\Rightarrow n = -\frac{JB}{eE_H}$, so we may determine n
- or $n = \frac{-1}{eR_H} \Rightarrow \frac{n}{n_{atom}} = \frac{-1}{eR_H n_{atom}}$ = ratio gives # charge carriers per atom

n_{atom} = # density of atoms per volume

→

Valency & n/n_{atom} should match:

material	valence	n/n_{atom} (measured)
Li	1	0.8
Na	1	1.2
K	1	1.1
Ca	2	1.5
Mg	2	-0.4

- the model does a fair job for valency 1, but valency 2 sometimes gives weird answers (like neg. values)

- we will find out later this tells us something about the charge carriers for metals like Mg ((+) holes carry charge)

- Recall that for Ohm's law, we could determine τn, for scattering time constant τ

- now we can solve for $\tau \approx 10^{-14}$ s for most metals at room T.

- one last note about the Hall effect: we can convert voltage to B-field as a Hall probe

$$B = -\frac{neE_H}{J} \quad \text{(field sensor)}$$

- let's think about a 1D metal

- the left side is hotter than the right $T_2 > T_1$
- heat crossing unit area per time

$$\underbrace{Q}_{\text{heat flux } (W/m^2)} = -K \frac{dT}{dx} \quad , \quad K \equiv \text{thermal conductivity}$$

- in metals, heat is conducted primarily by electrons since they are free to move
- if we again borrow from kinetic theory,

$$\underbrace{K}_{\text{Kappa}} = \frac{1}{3} n C_V \langle v \rangle \lambda \quad , \quad \underbrace{\lambda = \langle v \rangle \tau}_{\text{scattering length}}$$

(W/mK)

- for a monatomic gas $C_V = \frac{3}{2} k_B$

$$\langle v \rangle = \sqrt{\frac{8 k_B T}{\pi m}} \quad \longrightarrow$$

- average speed for a gas at T

- which gives us

$$K = \frac{4}{\pi} \frac{n \tau k_B^2 T}{m}$$

- we determined τ from Ohm's law
- if we look at the ratio of thermal to electrical conductivity

$$\sigma = \frac{e^2 \tau n}{m} \quad \text{+ } K \text{ from above}$$

- $L = \frac{K}{T\sigma} = \frac{4}{\pi}\left(\frac{k_B}{e}\right)^2 \sim 10^{-8} \frac{\text{Watt Ohm}}{K^2}$ (very close to measured values)
- Lorentz # 'L'
- another way to look at L

$$\frac{K}{\sigma} = LT \quad \text{(constant at fixed } T\text{)}$$

- matches experimental data for metals very well, but is wrong
- we don't get $C_v = \frac{3}{2} k_B$ for an electron

- specific heat ~ 100x too large
- but $\langle v \rangle$ ~ 100x too small

} cancel giving artificial correct answer

- if the electrons carry heat from the hot end to the cold end, then the gradient gives rise to an electrical current

- the Peltier effect exploits this as applying a potential to a thermoelectric material make's a refrigerator

- conversely, a thermocouple turns a temperature gradient into a voltage → thus you have a themometer (inexpensive + common)

- in 1925, Pauli discovered his "exclusion principle" that no two electrons may be in the exact same state

- Using this idea, we can describe the states of electrons in a conductor using Fermi-Dirac stats.

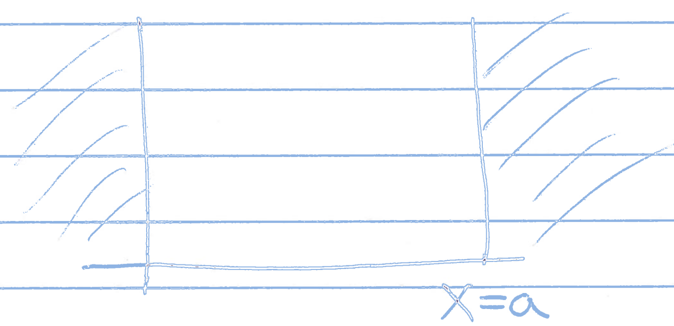
- Drude model assumed e^-'s can be in any state (classical) →

- let's do a little review of QM before moving into this discussion
- our time-ind. Schrödinger eq. in 1D

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

- recall the "particle in a box"

$$V(x) = \begin{cases} 0 & ; \quad 0 \le x \le a \\ \infty & ; \quad \text{otherwise} \end{cases}$$



- inside "box", $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$

$$\frac{d^2\psi}{dx^2} = -k^2\psi \, , \quad k \equiv \frac{\sqrt{2mE}}{\hbar}$$

$$\psi(x) = A\sin(kx) + B\cos(kx)$$

- boundary conditions:

$$\psi(0) = \psi(a) = 0$$

$$\psi(0) = A\sin(0) + B\cos(0) = 0$$

$$B = 0$$

$$\psi(a) = A\sin(ka) = 0$$

$$ka = 0, \pm\pi, \pm 2\pi, \ldots$$

$$k_n = \frac{n\pi}{a} = \frac{\sqrt{2mE_n}}{\hbar}$$

$$E_n = \frac{k_n^2 \hbar^2}{2m} \Rightarrow \boxed{E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}}$$

- normal modes with energy E_n
- after normalizing

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right)$$

- roughly, two metal atoms bond due to their free electrons having a lower energy in a larger "box" as $E_n \propto 1/a^3$