Mon. Sept. 10

- last time we began our discussion of Drude Theory

- We derived an equation for the average momentum of an electron in a conductor

$$\frac{d\vec{p}}{dt} = \vec{F} - \vec{p}/\tau$$

- recall $\vec{p} \to 0$ after a collison + τ is the mean free time between collisions

- Of course, the force applied to the electron gas is

$$\vec{F} = -e\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

or the Lorentz force

- the model is the same for the kinetic theory of gas, but gas is made from e^-'s, thus is Charged

- as a quick check, we ask what happens if $\vec{F} = 0$?

$$\frac{d\vec{p}}{dt} = \frac{\vec{p}}{\tau}$$

$$\Rightarrow \vec{p}(t) = \vec{p}_0 e^{-t/\tau}$$, as expected

- we stated that after a collision $\vec{p} \to 0$, so on average $\vec{p} \to 0$ (decays) with characteristic time τ

- recall for exp. decay

$$@\, t = \tau, \quad \frac{|\vec{p}|}{|\vec{p}_0|} = e^{-1} \simeq 1/3$$

- decay constant tells us how fast average $\vec{p}$ decays to e^{-1} of the original value

- that is a good check for consistency (i.e. when $\vec{F} = 0$, $\vec{p}$ decays), but what we want to investigate is of course $\vec{F} \neq 0$, so $\vec{E}$ +/or $\vec{B}$-fields are present

- let's look at an E-field only
- typical example, attach ends of a conductor to battery terminals

$$\frac{d\vec{p}}{dt} = -e\vec{E} - \frac{\vec{p}}{\tau}$$

- at steady state, driving voltage (E-field) + drag balance so

$$\frac{d\vec{p}}{dt} = 0 \quad \text{(no accel.)}$$

$\Rightarrow$ this is a good approximation to reality due to scattering

- thus we have

$$\vec{p} = -e\tau\vec{E}$$

- recall volume current density is

$$\vec{J} = \rho\vec{v}, \quad \rho \equiv \text{charge}/\text{vol}$$

- thus J has units $\frac{C}{m^3}\cdot\frac{m}{s} = \frac{C}{s}\left(\frac{1}{m^2}\right)$

- which of course is current/area, where area is cross-section

- For volume # density of electrons, n,

$$\rho = -en \quad + \quad \vec{J} = -en\vec{v}$$

- our equation for momentum gave

$$\vec{p} = -e\tau\vec{E} = m\vec{v} \quad (\text{steady state } \vec{v})$$

$$\Rightarrow \vec{v} = \frac{-e\tau\vec{E}}{m}$$

$$\Rightarrow \quad \vec{J} = -en\left(\frac{-e\tau}{m}\right)\vec{E}$$

- recall Ohm's law

$$\vec{J} = \sigma\vec{E}, \quad \sigma \equiv \text{conductivity}$$

- thus $\sigma = \dfrac{e^2\tau n}{m}$

- We know what e + m are, charge + mass of e^-

$\rightarrow$

- so the theory makes a prediction:

when we go to the lab + measure σ by applying voltages (E-field) + measuring current we use

$$\sigma = \frac{e^2 \tau n}{m} \Rightarrow \frac{\sigma m}{e^2} = \tau n$$

- measure σ, we know m + e, now we can find τn
- if we know what n, the density of electrons, we can find τ
- how do we know n?
- let's apply a B-field to the system also
- the eq. of motion is slightly more complex

$$\frac{d\vec{p}}{dt} = -e(\vec{E} + \vec{v} \times \vec{B}) - \vec{p}/\tau$$

- again assume steady state $\frac{d\vec{p}}{dt} = 0$

- with $\vec{p} = m\vec{v}$ + $\vec{J} = -ne\vec{v}$

$$\vec{p} = m\left(\frac{-\vec{J}}{ne}\right), \quad \vec{v} = \frac{-\vec{J}}{ne}$$

- $$0 = -e\vec{E} + \frac{\vec{J}\times\vec{B}}{n} + \frac{m}{ne\tau}\vec{J}$$

$$\Rightarrow \quad \vec{E} = \left(\frac{1}{ne}\vec{J}\times\vec{B} + \frac{m}{ne^2\tau}\vec{J}\right)$$

- this equation can be expressed as a "resistivity matrix" $\underset{\sim}{\rho}$

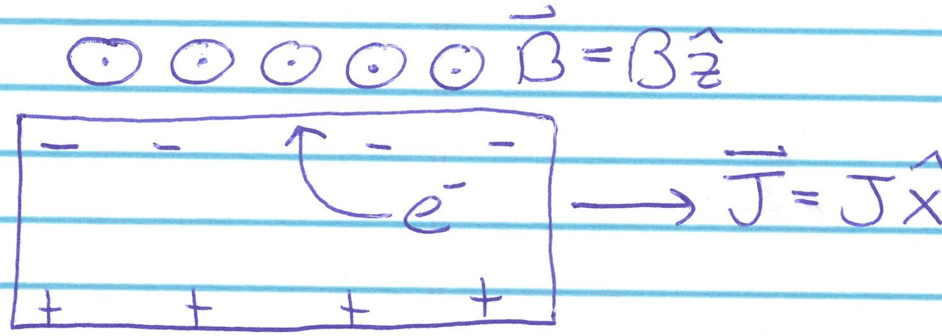
so $\vec{E} = \underset{\sim}{\rho}\,\vec{J}$ (compare to $\vec{J} = \sigma\vec{E}$ ohm's law)

- let's look at a particular example of the "Hall effect", first term in eq. above (circled)

Over

$\longrightarrow$

- take a conductive "slab" and run current $\vec{J}$ through conductor & apply a $\perp$ $\vec{B}$-field (uniform)



- We know from the Lorentz force $\vec{F} = -e(\vec{v} \times \vec{B})$ the electrons feel a force

$$\vec{v} \times \vec{B} \rightarrow v\hat{x} \times B\hat{z} = vB(-\hat{y})$$

- due to the $(-)$ sign, e^-'s feel a force upward $(+\hat{y})$

- $(+)$ charges are an absense of electrons

- We define the Hall field

$$\vec{F}_H = e\vec{E}_H = eE_H(\hat{y}) \text{ for our problem}$$

& lorentz $\vec{F}_L = F_L(-\hat{y})$

- at steady state,

$$eE_H = evB$$

- recall $\vec{J} = (-ne)\vec{v}$

ρ

$\Rightarrow E_H = \left(\frac{-J}{ne}\right)B = \left(\frac{-1}{ne}\right)\vec{J}B$

- & define $R_H = \left(\frac{-1}{ne}\right) \equiv$ "Hall coeff."

- Why does this matter?

- we know e, so supply current & B-field & measure voltage → or E_H

- now we know $n = \frac{N}{vol}$, charge carrier density