Fri. Sept. 07

- today we will finish our discussion of Debye's theory

- to recap from last time, the expectation value of energy for a QHO

$$\langle E \rangle = \frac{1}{Z} \sum_n E_n e^{-\beta E_n}$$

$$= \frac{\hbar\omega}{e^{\beta\hbar\omega} - 1}$$

- to find the total energy of a solid, we multiply $\langle E \rangle$ by $g(\omega)d\omega$ and integrate

$$E = \int_0^\infty \langle E \rangle g(\omega) d\omega$$

- $$E = \frac{3V}{2\pi^2 v_s^3} \int_0^\infty \frac{\omega^2 \hbar\omega}{e^{\beta\hbar\omega} - 1} d\omega$$

- total number of degrees of freedom $= 3N$

$$\int_0^{\omega_d} g(\omega)\,d\omega = 3N \quad , \quad N \equiv \#\text{ atoms}$$

$\omega_d \equiv$ Debye Frequency

- Solving the integral gives

$$\omega_d = (6\pi^2 n)^{1/3} \quad , \quad n = N/V \text{ (density)}$$

$$C_V = \frac{\partial E}{\partial T} \quad , \text{ where } E = \frac{3V}{2\pi^2 v_s^3} \int_0^{\omega_d} \frac{\hbar\omega^3}{e^{\beta\hbar\omega} - 1}\,d\omega$$

$$\Rightarrow C_V = \frac{3V}{2\pi^2 v_s^3} \frac{\hbar^2}{k_B T^2} \int_0^{\omega_d} \frac{\omega^4 e^{\beta\hbar\omega}}{(e^{\beta\hbar\omega} - 1)} d\omega$$

- let $x = \frac{\hbar\omega}{k_B T} = \beta\hbar\omega$

$$C_V = 9R \left(\frac{T}{T_D}\right)^3 \int_0^{T_D/T} \frac{x^4 e^x}{(e^x - 1)^2}\,dx$$

$\rightarrow$ some #

$C_V \sim T^3$, matching low T behavior //

- Some limitations of Debye's theory:

(1) $\omega = vk$ does not always hold

(2) only accurate for low T

(3) Cannot account for $C_v \propto T$ for very low T in metals

- For metals, electrons are "free" & contribute to C by gaining energy from added heat

- We see that $C_v = \gamma T + \alpha T^3$ for metals shows dominance in the linear term when T is very small as $T^3 \to 0$ for $T \ll 1$

- At this point, we could move on to correct Debye's theory or we could look at how electrons contribute to C_v

- in accordance with the book, we will do the latter //

- So far in this class we have neglected the other major component of solids, the electron

- We first study a classical theory of metals called Drude theory

- We don't ask the question of why metals are conductive (yet), but what are the properties?

- the theory is remarkably simple (or its foundation is) as it utilizes kinetic theory of gas but the gas is charged (responds to fields)

- Drude theory makes 3 assumptions

(1) the probability of scattering in a time ~~[illegible]~~ dt is dt/τ, where

τ is the scattering time (mean free path time)

(2) after a scattering event, the electron returns to $\vec{p} = 0$

(3) with charge $-e$, the "gas" responds to $\vec{E}$ & $\vec{B}$-fields

- (1) + (2) come directly from Boltzmann's theory of gas (kinetic theory) $\langle \frac{1}{2}mv^2 \rangle = \frac{3}{2}k_B T$

- it is possible to calculate τ for neutral gas, but the calculation is complex for charged particles

- the particles returning to $\vec{p} = 0$ seems dubious, and is, but conserving $\vec{p}$ makes calculation much more complex

- we start by assigning an electron in the gas with a momentum $\vec{p}$ at time t

- What is $\vec{p}$ at $t + dt$

- probability dt/τ scattering occurs + $\vec{p} = 0$

- if no scattering, e^- continues to accelerates

$$\frac{d\vec{p}}{dt} = \vec{F}$$

- when we combine these, we may obtain the equation →

- we get:

$$\langle \vec{p}(t+dt) \rangle$$

$$= \left(1 - \frac{dt}{\tau}\right)\left(\vec{p} + \vec{F}dt\right) + 0\frac{dt}{\tau}$$

$\vec{p} \to 0$

prob. of not scattering

Force applied during dt

prob. of scattering

- note that $\vec{p}$ is the thermal average $\langle \dots \rangle$
- drop brackets $\langle \dots \rangle$
- $\vec{p}(t+dt) = \vec{p}(t) + \vec{F}dt - \frac{dt}{\tau}\vec{p} - \vec{F}\frac{dt^2}{\tau}$
- ignoring $(dt)^2$ term → very small)

$$\vec{p}(t+dt) - \vec{p}(t) = \vec{F}dt - \frac{dt}{\tau}\vec{p}$$

- divide by dt

$$\Rightarrow d\vec{p}/dt = \vec{F} - \vec{p}/\tau$$

- we may interpret the $\frac{\vec{p}}{\tau}$ as a "drag" force from interactions with the lattice

- What do we expect to happen in the absense of a force?

$$\frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau}$$

$$\vec{p}(t) = \vec{p}_0 e^{-t/\tau}, \text{ as expected}$$

which means momentum decays exponentially, as was built into the model

- the EM force is $\vec{F} = -e(\vec{E} + \vec{v} \times \vec{B})$
- let's look at $\vec{E}$-field only
- at steady state $\frac{d\vec{p}}{dt} = 0$

$$\frac{d\vec{p}}{dt} = -e\vec{E} - \frac{\vec{p}}{\tau} = 0 \Rightarrow$$

$$\vec{p} = -e\tau\vec{E} = m\vec{v}$$

- current is given by

$$\vec{J} = -en\vec{v}, \quad n = e^{-}\text{-density}$$

- recall $J \rightarrow \frac{C}{m^2}$ $\qquad \rho \rightarrow C/m^2$

$$\vec{J} = -en\left(\frac{-e\tau}{m}\right)\vec{E}$$

- recall Ohm's law

$$\vec{J} = \sigma\vec{E}$$

so $\sigma = \frac{e^2\tau n}{m}$, is the conductivity of the medium

- since we know e^{-} + m, we can determine τn by measuring σ (in the lab)

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Debye vs Einstein model for specific heat of silver