Wed. Oct. 24

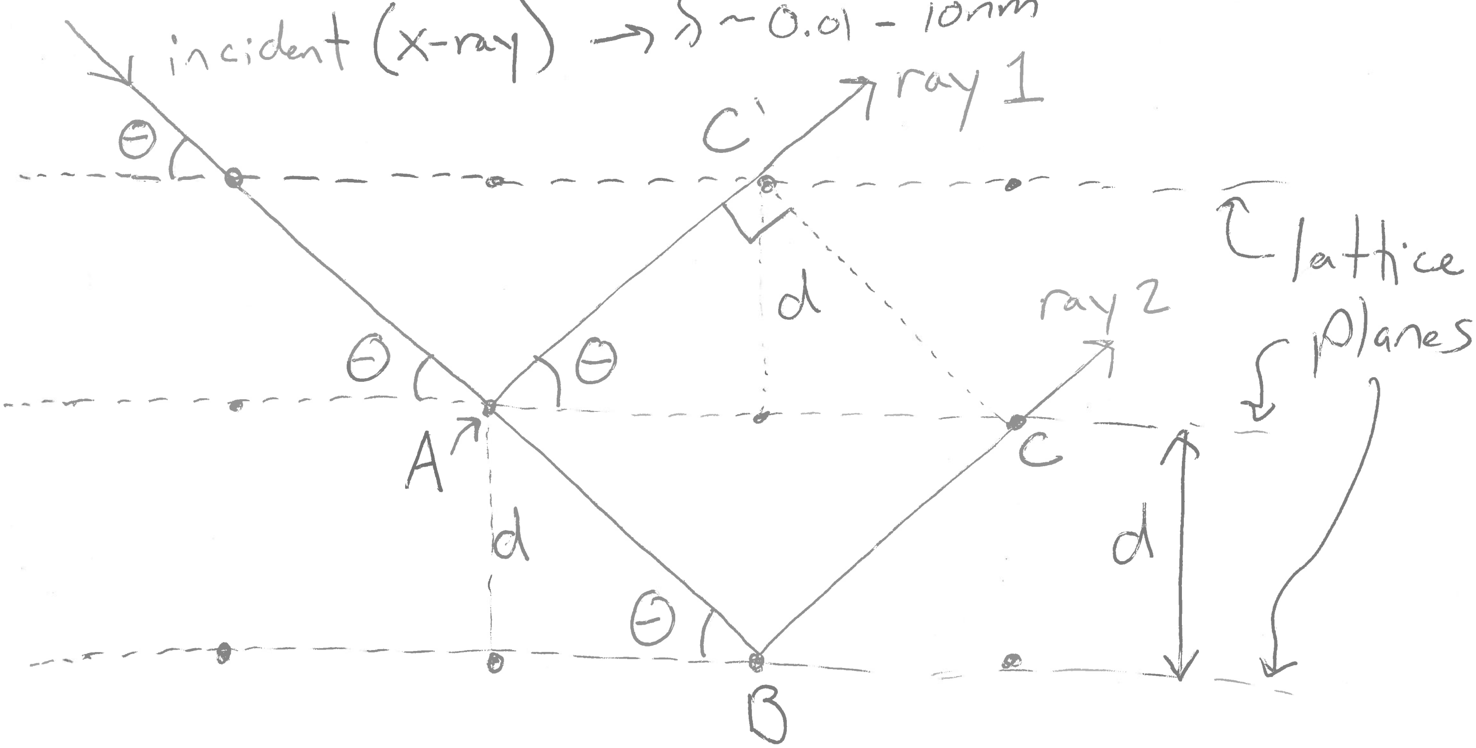
- We have been studying families of planes in crystals
- Now we want to put this knowledge to use
  → Wave scattering by crystals
- We know that when the path diffence, Δ, between two rays of light with wavelength, λ, constructively interfere when

$$\Delta = n\lambda$$

where n = integer

- We recognize crystal planes reflect x-rays + are partially transparent
- Constructive interference is a function of plane spacing, d, λ, + θ, the angle of incidence : $n\lambda = 2d \sin\theta$

• consider the following geometry:



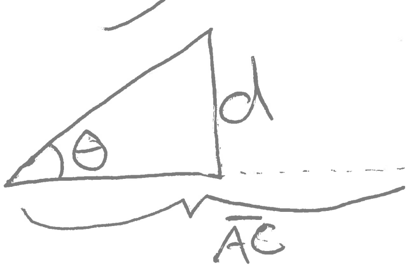
• What is the path difference? between ray 1 + 2

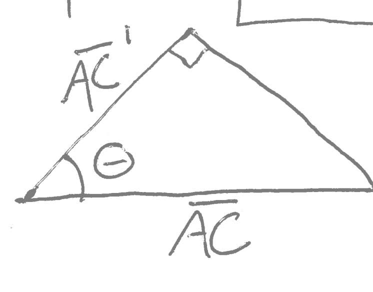
$$\Delta = \overline{AB} + \overline{BC} - \overline{AC'} = 2\overline{AB} - \overline{AC'}$$

• note $\overline{AB} = \overline{BC}$ (no refraction)

• note: $\sin\theta = \dfrac{d}{\overline{AB}}$ + $\cos\theta = \dfrac{\overline{AC'}}{\overline{AC}}$

→ $\tan\theta = \dfrac{d}{\frac{1}{2}\overline{AC}}$





- now we have

$$\overline{AC}' = \overline{AC}\cos\Theta = \underbrace{\left(\frac{2d}{\tan\Theta}\right)}_{\overline{AC}} \cdot \cos\Theta$$

- now let's calculate $\Delta = 2\overline{AB} - \overline{AC}'$

$$2\overline{AB} = \frac{2d}{\sin\Theta}, \quad \overline{AC}' = \frac{2d}{\frac{\sin\Theta}{\cos\Theta}} \cdot \cos\Theta = \frac{\cos^2\Theta\, 2d}{\sin\Theta}$$

$$\Rightarrow \Delta = \frac{2d}{\sin\Theta}\Big[\underbrace{1 - \cos^2\Theta}_{\sin^2\Theta}\Big]$$

$$\Rightarrow \Delta = \boxed{2d\sin\Theta = n\lambda}$$ when interference is constructive

- thus showing the form of "Bragg's law"

- A few notes about Bragg's law:

(1) Reflection only occurs when $\Delta = n\lambda$, all other waves pass through crystal undisturbed (where have we seen this effect before?)

(2) We can clearly determine $d = \frac{n\lambda}{2\sin\theta}$ from experiments

(3) diffraction only occurs for $\lambda \leq 2d$, so high energy (x-rays) need to be used $\rightarrow$ diffraction limit

* smallest $n = 1$

$$\Rightarrow \frac{\lambda}{2d} = \sin\theta; \quad -1 \leq \sin\theta \leq 1$$

$$\Rightarrow \frac{\lambda}{2d} = \sin\theta \leq 1 \Rightarrow \lambda \leq 2d$$

(4) this law is an approximation in this case as we have not considered atomic makeup

- in reality the incident wave (of x-rays) scatters from the electrons in the crystal
- the details should be considered, but we will only mention them here due to time

Ex:

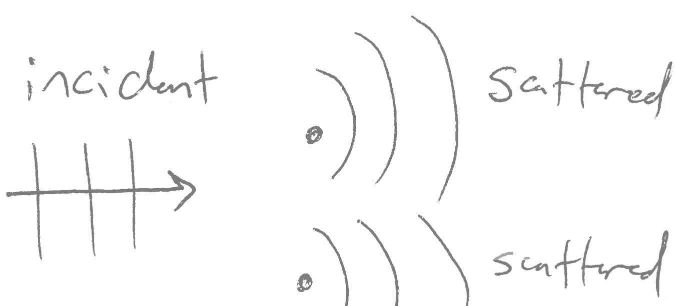
$$u = Ae^{i(\vec{k}_0 \cdot \vec{r} - \omega t)}$$

$$u' = f_e \frac{A}{D} e^{i(kD - \omega t)}$$

f_e = scattering length of e^-

$D \rightarrow$ distance from electron

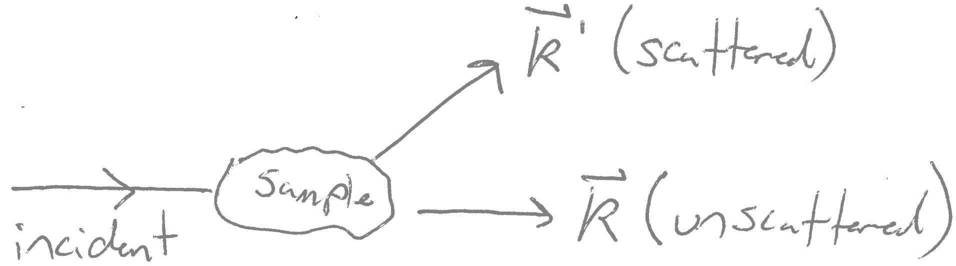
- for two electrons, the scattering fields interfere



$$u' = f_e \frac{A}{D}\left[e^{ikD} + e^{i(kD+\delta)}\right]$$

δ = phase diff. (l.g.)

Laue Condition:



- transition rate (per unit time)

$\rho \to$ density of final states

$$\Gamma(\vec{k}', \vec{k}) = \frac{2\pi}{\hbar} \left| \langle k' | V | k \rangle \right|^2 \delta(E_{k'} - E_k)$$

$\to$ Fermi's golden rule

- $V(\vec{r}) \to$ potential of the sample

$$\langle \vec{k}' | V(\vec{r}) | \vec{k} \rangle = \int d\vec{r} \, \frac{e^{-i\vec{k}'\cdot\vec{r}}}{\sqrt{L^3}} \frac{e^{i\vec{k}\cdot\vec{r}}}{\sqrt{L^3}} V(\vec{r})$$

matrix element

$$= \frac{1}{L^3} \int d\vec{r} \, e^{-i(\vec{k}'-\vec{k})\cdot\vec{r}} \, V(\vec{r})$$

$\to$ this is the Fourier transform of $V(\vec{r})$