Mon. Oct. 22

- We have been discussing ordering in solids, lattices, + reciprocal lattices

- recall the reciprocal lattice is the Fourier transform of the real lattice

- another way to visunlize reciprocal lattice is via lattice planes

- Lattice plane: a plane that contains 3 non-colinear lattice points

- Family of lattice planes: equally spaced parallel lattice planes that intersect all points of the lattice

- see slides for examples

→

- there is a nice connection between the families of lattice planes & reciprocal lattice
- the reciprocal lattice vectors are normal to the families of lattice planes
- further, the spacing between planes is given by

$$d = 2\pi / |\vec{G}_{min}|$$

- where $\vec{G}_{min}$ is the minimum length rec. lattice vector in the normal direction
- Check 1 D case:

$$\text{real:} \quad \overbrace{-2a \qquad -a}^{d} \qquad 0 \qquad a \qquad 2a$$

$$\text{reciprocal:} \quad \underbrace{-2(2\pi/a) \qquad -2\pi/a}_{2\pi/a} \qquad 0 \qquad 2\pi/a \qquad 4\pi/a$$

G_{min}

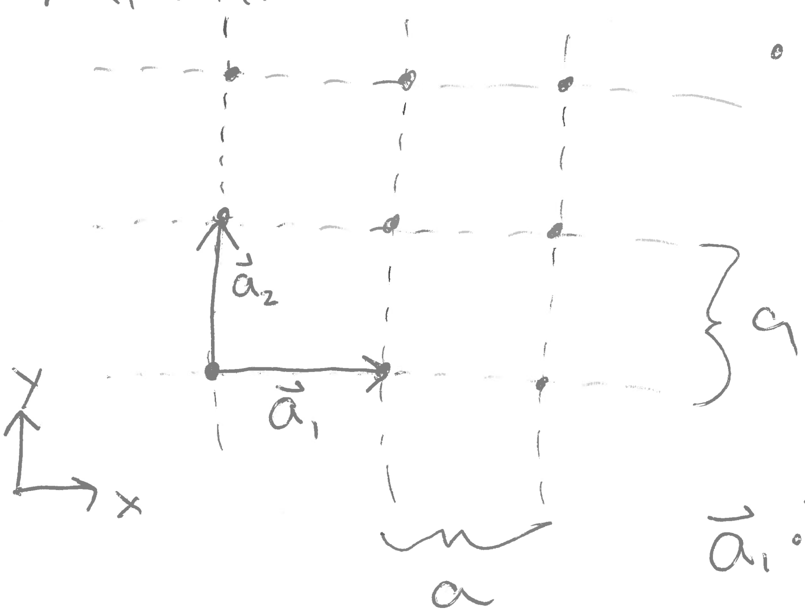
• spacing between planes (points)

$$d = 2\pi / G_{min} = 2\pi / (2\pi/a) = a$$

as expected

• Scaling up → 2D Square lattice

real lattice



• dashed lines = families of planes

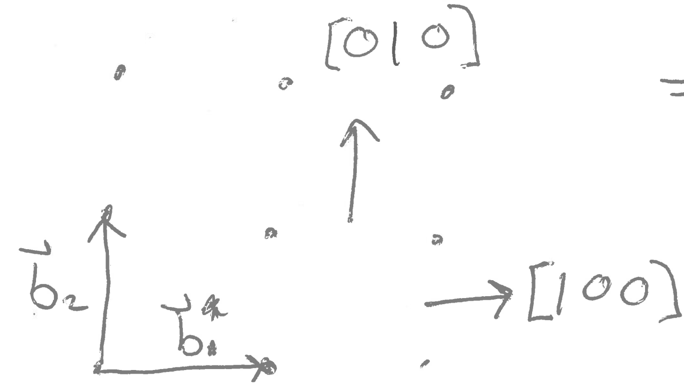
$$e^{i\vec{R}\cdot\vec{G}} = 2\pi$$

$\vec{a}_1 \cdot \vec{b}_1 = 2\pi$, by definition

$|\vec{a}_1| = a$ + $|\vec{b}_1| = 2\pi/a$

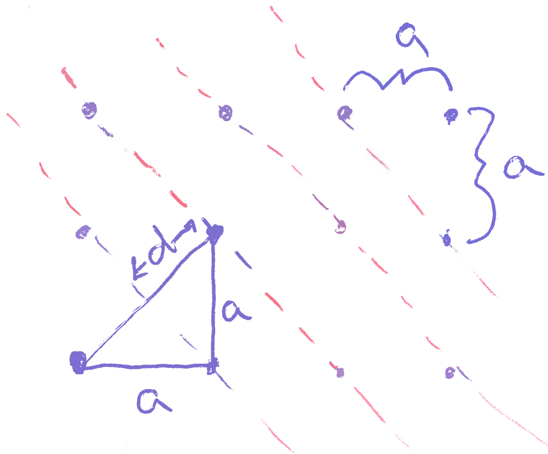
$\Rightarrow \vec{a}_1 \cdot \vec{b}_1 = 2\pi$ ✓

recip. lattice



$$d = \frac{2\pi}{|G_1|} = 2\pi / (2\pi/a) = a$$ ✓

- How about diagonal direction?



- We expect

$$d = \frac{1}{2}\sqrt{a^2 + a^2} = \frac{a\sqrt{2}}{2}$$

$\nearrow$ [1 1 0] direction

- the direction is given by $[1\ 1\ 0] = [h\ k\ \ell]$ defined by $\vec{G} = h\vec{b}_1 + k\vec{b}_2 + \ell\vec{b}_3$

- for 3D, the separation is given by

$$d = \frac{2\pi}{|\vec{G}|} = \frac{2\pi}{\sqrt{h^2|\vec{b}_1|^2 + k^2|\vec{b}_2|^2 + \ell^2|\vec{b}_3|^2}}$$

$1^2 \quad \left(\frac{2\pi}{a}\right)^2 \quad 1^2 \quad \left(2\pi/a\right)^2 \quad 0$

$$d = \frac{2\pi}{\sqrt{1^2\left(\frac{2\pi}{a}\right)^2 \cdot 2}} = \frac{2\pi}{\frac{2\pi}{a}\sqrt{2}} = \frac{a}{\sqrt{2}} = \frac{a\sqrt{2}}{2} \checkmark$$