Fri. Sept. 21

- last time we showed the total energy from the electrons in a free electron gas leads to a quantum pressure (see Wed. lecture)

$$E_{tot} = \frac{\hbar^2 (3\pi^2 N q)^{5/3} V^{-2/3}}{10\pi^2 m}$$

- recall we are discussing a solid of dimension (ℓ_x, ℓ_y, ℓ_z)

- let's assume the solid expands in volume by dV, what change in E_{tot} do we get?

$$\frac{dE_{tot}}{dV} = \frac{-2}{3} \frac{\hbar^2 (3\pi^2 N q)^{5/3}}{10\pi^2 m} V^{-5/3}$$

$$V^{-5/3} \rightarrow \frac{V^{-2/3}}{V}$$

$\Rightarrow dE_{tot} = \frac{-2}{3} E_{tot} \frac{dV}{V}$, so energy drops as expected

- this change in energy shows up as work done on the outside, dW

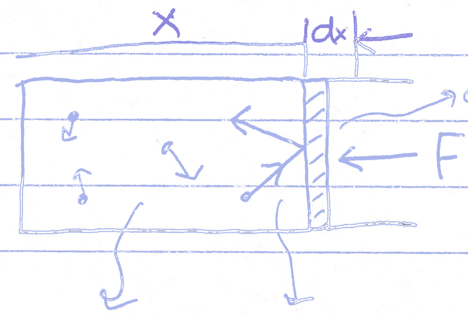
$$dW = -dE_{tot} \quad \text{(work drops energy)}$$

$\Rightarrow dW = PdV = -dE_{tot} = \frac{2}{3} E_{tot} \frac{dV}{V}$ $\rightarrow$

- thus, $P = \frac{2}{3} E_{tot}/V \propto \rho^{5/3}$

- this result makes sense intuitively: higher internal energy or lower volume means higher pressure

- recall from the kinetic theory of gases:



$$dW = F(-dx) = -PA\,dx = -P\,dV$$

volume V collisions with movable barrier supply counter force

- atoms have kinetic energy, so relate KE to P

- from kinetic theory, we get $PV = \frac{2}{3} E_{tot}$ the exact same formulation as quantum pressure

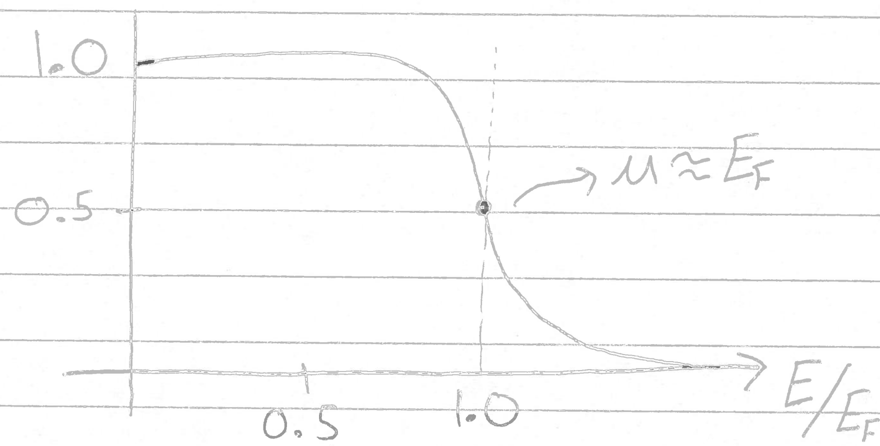
- two distinct mechanisms give rise to the same result: the internal energy applies an outward pressure //

- statistically, we may define the Fermi-Dirac distribution as

$$n_F = \left(e^{\beta(E-\mu)}+1\right)^{-1}$$

where $\mu \equiv$ chemical potential

- the chemical potential is equal to the Fermi energy at $T=0K$



- curve for $T>0$

- $n=1$, states are occupied
$n=0$, states are empty

- let's look at $T=0$:

$$E<0 \Rightarrow E-\mu<0$$

exponent argument $<0 \Rightarrow n_F = 1$
(for $E<\mu$)

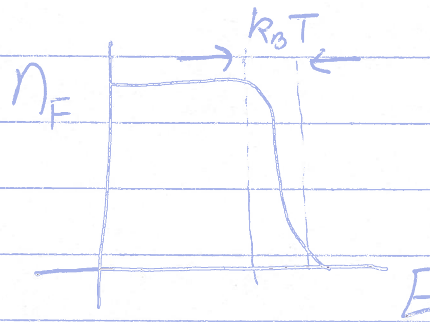
- similarly, also for $T=0$, denominator $\rightarrow \infty$, so $n_F = 0$ (states empty)

- we see that $n_F = \begin{cases} 1, & E<\mu \\ 0, & E>\mu \end{cases}$

or a step function

- for $T>0$, some of the most energetic electrons move into excited states $E>E_F \sim \mu$, leaving unoccupied states for $E<E_F$

- unsurprisingly, the spread near μ is on the order of the thermal energy $k_B T$

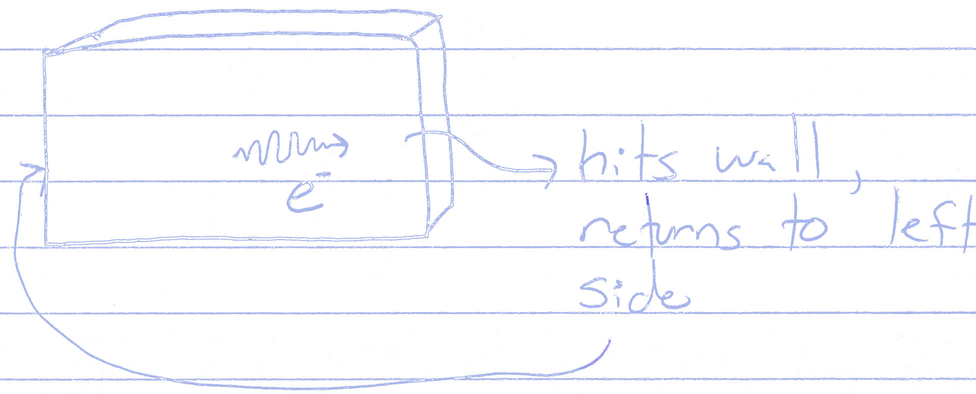


- due to thermal excitations

- higher T, more spreading

- we utilized "hard wall" boundary conditions to solve for the states of electrons in a conductor

- using periodic boundary conditions, we get the same answer, but interpretation is slightly different (will get to later)



- let's look at an approximation for the heat capacity of electrons

- we know the low energy states cannot absorb heat because energy levels above are occupied

- the calculation is quite involved, so we will approximate

- we know only only electrons near E_F can be excited, and the fraction of electrons capable of absorbing heat

$$\sim \frac{kT}{E_F}$$

- if the fraction of electrons able to absorb heat is $\frac{k_B T}{E_F}$

and each electron may absorb $\sim k_B T$ of energy, the total energy

$$\sim \left(\frac{k_B T}{E_F}\right) k_B T$$

so $C_V^{e^-} = \frac{\partial E}{\partial T} \simeq \frac{2k_B^2}{E_F} T$ or αT

alpha

- experimentally, we found $C_V = \alpha T + \gamma T^3$

& so at very low T, the linear term dominates

- thus we have determined the origin of this linear term for C_V in metals //

- we realize the Fermi energy corresponds to Fermi temp. and speed

- choose lithium (Li), with $E_F \simeq 5eV$

we get $T_F \approx 60000K$

- we would need to raise the temp. of Li to around T_F to recover the classical value of C_V, which cannot be done

- What about the speed V_F?

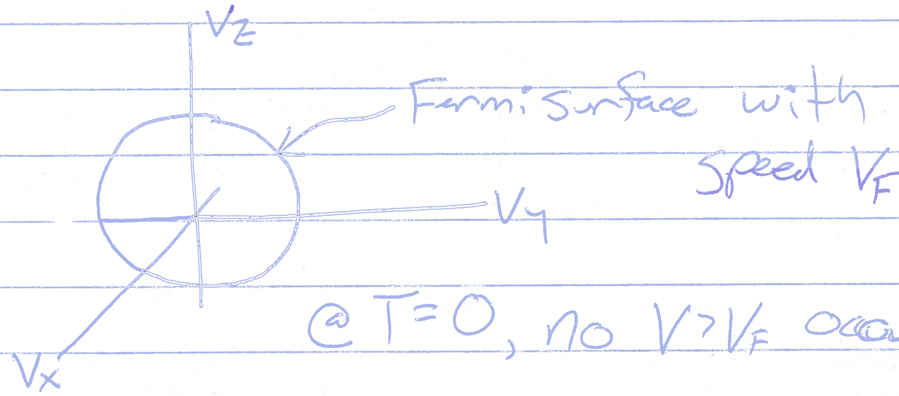
- we calculate $V_F \approx 10^6 m/s$, $\sim 0.01c$ speed of light

$$E_F = \frac{1}{2} m V_F^2$$

even at $K=0$!

- the Fermi Surface

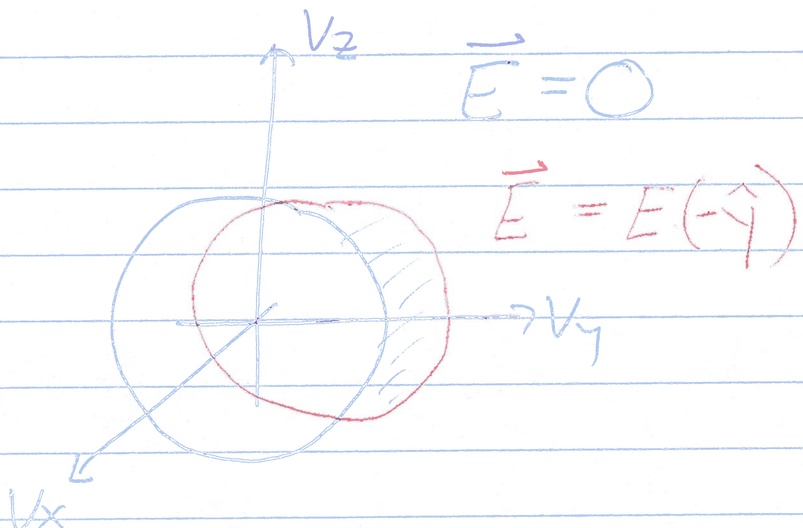
- if we plot speeds with direction in velocity space



@ $T=0$, no $V > V_F$ occupied

- each point inside sphere is a single e^- with specific $\vec{V} = (v_x, v_y, v_z)$

- electrical current can be visualized this way: @ $T = 0$



- We see that only a small fraction of high speed electrons contribute to current (shaded region), not all, as was assumed in Drude Model

- Limitations of free electron gas

$$\lambda = V_F \tau \sim 100\text{Å} \sim 100 \times \text{atomic spacing}$$

- core electrons do not "count" for calculating Fermi level

- Sign of charge carriers sometimes < 0
- plasma frequency of metals is different
- cannot explain insulators / semi cond.