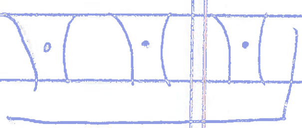
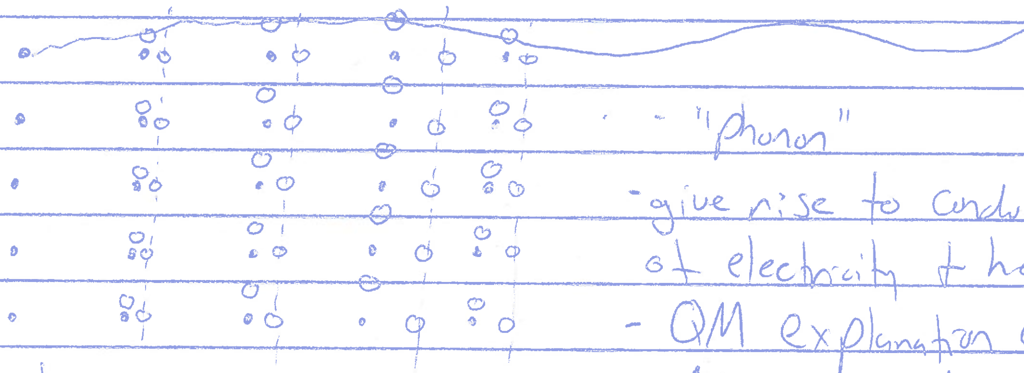
- in a solid: elastic arrangement of atoms



potential well



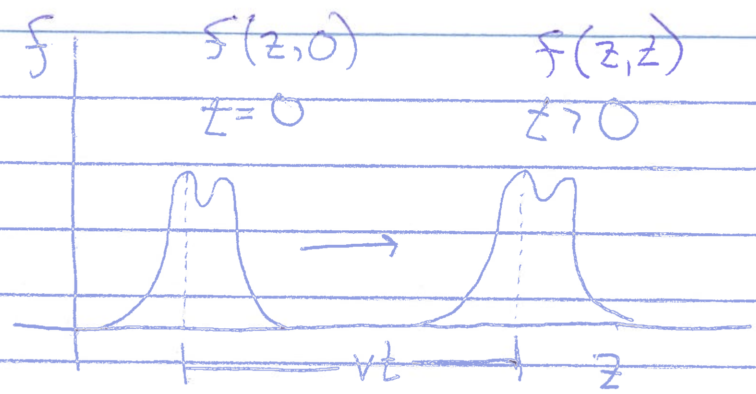
- "phonon"
- give rise to conductivity of electricity + heat
- QM explanation of vibrational modes

- transverse ↕

- longitudinal ⟷

- waves exist in many forms
- mechanical wave - disturbances moving through matter that transfers energy
- Mass has no net displacement in this case

ex: - Schrödinger eq. explains wave-behavior of particles
- Gravitational wave - predicted by Gen. Rel. but have never been observed. (new resluts but dubious)
  - binary system will radiate gravitational radiation
  - like in E+M, charges radiate EM radiation, accelerating masses radiate Grav. radiation
  - effect is small except for closely orbiting, very massive bodies (like black holes)
  - Earth's orbit is reduced every year
    Q: by 1/300 diameter of a hydrogen atom
- We are interested in E+M waves
- can move without medium,
- all waves have similarities + can be described mathematically



- constant speed to the right

- disturbance in medium (string) maintains its shape wrt time, but position on the z-axis changes
- in this example - pulse is moving to the right

- assume initial shape is $g(z) \equiv f(z, t=0)$
- shape is the same at a later time, f is identical to the function at $t=0$ if the function is shifted $z-vt$ to the left

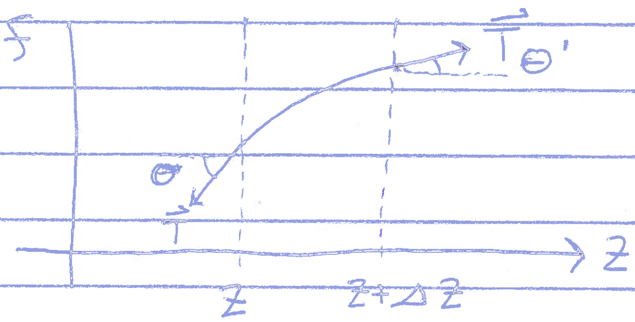
$$\Rightarrow f(z,t) = f(z-vt, 0) = g(z-vt)$$

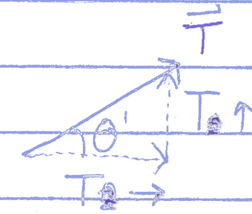
- function must depend on z + t in the form $z-vt$ to describe a wave moving to the right

Q: - $f_1(z,t) = Ae^{-b(z-vt)^2}$, $f_2(z,t) = Ae^{-b(bz^2+vt)}$

- which is a wave moving toward the right?

- Wave Theory: wave on a string model





- this is a transverse wave - motion perpendicular to the direction of travel $\rightarrow +\hat{z}$

$$T\sin\theta' - T\sin\theta = \Delta F\,(\text{transverse})$$

- assume angle θ is small

$$\Delta F \approx T\tan\theta' - T\tan\theta \quad \rightarrow$$

hyp. & adj close to same

$$\Delta F = T\left(\frac{\partial f}{\partial z}\bigg|_{z+\Delta z} - \frac{\partial f}{\partial z}\bigg|_{z}\right)$$

$$\lim_{\Delta z\to 0}\frac{1}{\Delta z}\left(\frac{\partial f}{\partial z}\bigg|_{z+\Delta z} - \frac{\partial f}{\partial z}\bigg|_{z}\right) = \frac{\partial^2 f}{\partial z^2} \Rightarrow \Delta F = \frac{\partial^2 f}{\partial z^2}\Delta z\, T$$

- second law:

$$\Rightarrow \Delta F = \underbrace{\mu(\Delta z)}_{m}\ \underbrace{\frac{\partial^2 f}{\partial t^2}}_{a} \quad ; \quad \mu \equiv \text{mass/length}$$

$$\Delta F = \frac{\partial^2 f}{\partial z^2} - \underbrace{\frac{\mu}{T}}_{1/v^2}\frac{\partial^2 f}{\partial t^2} \rightarrow v = \sqrt{\frac{T}{\mu}}$$

- speed of propagation

- all functions of the form

$$f(z,t) = g(z-vt) \text{ satisfy } \frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

- most general solution is actually

$$f(z,t) = g(z-vt) + h(z+vt)$$

moving to the left

- why? b/c wave equation is linear $\Leftrightarrow$ sum of any 2 solutions is also a solution

- $g(u)$ can be any differentiable function if it does not change shape.

- $f(z,t) = A\sin(kz)\cos(kvt)$

$$\frac{\partial^2 f}{\partial z^2} = -Ak^2\sin(kz)\cos(kvt)$$

$$\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = -Ak^2\sin(kz)\cos(kvt)$$

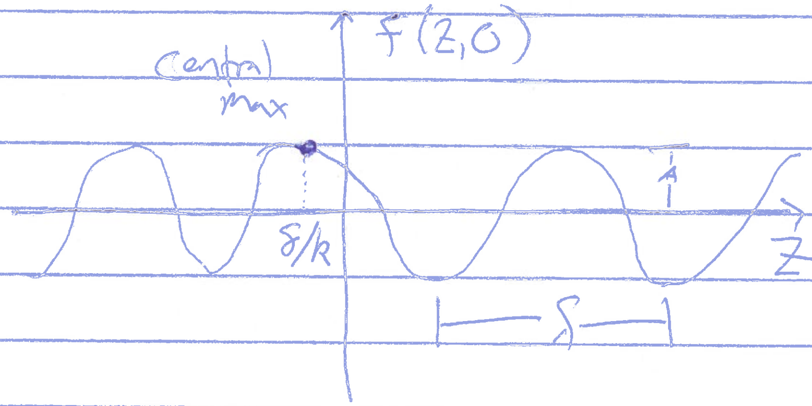
- satisfy's wave eq. ✓

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$f(z,t) = \frac{A}{2}\left[\sin(k[z+vt]) + \sin(k[z+vt])\right]$$

- standing wave

· Sinusoidal wave: $f(z,t) = A\cos[(k(z-vt))+\delta]$



$k(z-vt)+\delta \rightarrow$ phase of wave

$\delta \equiv$ phase constant (delay constant) $\Rightarrow z = vt - \delta/k$ phase $= 0$ ($\delta/k \rightarrow$ dist. delayed)

$k \equiv \frac{2\pi}{\lambda} \equiv$ wave number, $T = \frac{2\pi}{kv}$ $v \rightarrow$ velocity (speed)

$\nu = \frac{1}{T} = \frac{kv}{2\pi} = \frac{v}{\lambda}$, $\omega = 2\pi\nu = kv$

$\Rightarrow f(z,t) = A\cos(kz - \omega t + \delta) \rightarrow$ to the right

$f(z,t) = A\cos(kz + \omega t - \delta) \rightarrow$ to the left

$-\delta$ (minus) because $\frac{\delta}{k}$ is the distance delay.

- cos is even $\Rightarrow \cos(x) = \cos(-x)$

$\rightarrow f(z,t) = A\cos(-kz - \omega t + \delta)$ - switch k, wave traveling in opposite dir.

- Complex notation:

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \text{(Euler's formula)}$$

$$f(z,t) = \mathrm{Re}\left[A e^{i(kz-\omega t+\delta)}\right]$$

$$= A\cos(kz-\omega t+\delta)$$

$$\tilde{f}(z,t) \equiv \tilde{A} e^{i(kz-\omega t)}, \quad \tilde{A} \equiv A e^{i\delta}$$

- complex wave function — complex amplitude

- the actual wave is $f(z,t) = R\left[\tilde{f}(z,t)\right]$

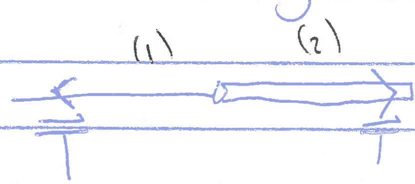
- Any wave can be expressed as a linear combination of sinusoidal waves:

end Friday Nov. 14th

$$\tilde{f}(z,t) = \int_{-\infty}^{+\infty} \tilde{A}(k) e^{i(kz-\omega t)}\, dk \quad \text{(-k means both directions)}$$

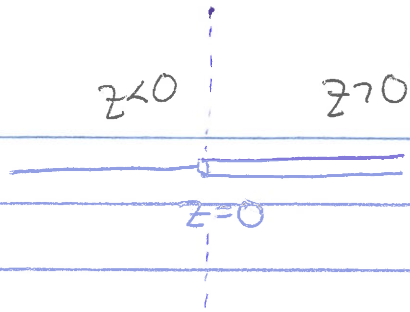
- Boundary Conditions:

- Consider 2 strings of different $\mu \equiv m/\ell$



tension is the same

velocity of impulse is different $v = \sqrt{\frac{T}{\mu}}$



$e^{i\theta} = \cos\theta + i\sin\theta$

$\tilde{A}_I = A_I e^{i\delta}$, $\omega = k_1 v_1$

- wave from the left: $\tilde{f}_I(z,t) = \tilde{A}_I e^{i(k_1 z - \omega t)}$; $z<0$
- reflected wave: $\tilde{f}_R(z,t) = \tilde{A}_R e^{i(-k_1 z - \omega t)}$; $z<0$

$\omega = k_2 v_2$

- transmitted wave: $\tilde{f}_T(z,t) = \tilde{A}_T e^{i(k_2 z - \omega t)}$; $z>0$

- the frequency, ω, is the same because this comes from the source.

$v = \lambda \nu$ $\qquad$ $n = \frac{c}{v}$

$\lambda = 2\pi/k$ $\qquad$ $\frac{v_1}{v_2}$

$\lambda = \frac{2\pi}{k} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{k_2}{k_1} = \frac{v_1}{v_2}$ $\left(\omega = k_1 v_1 = k_2 v_2 \Rightarrow \frac{k_2}{k_1} = \frac{v_1}{v_2} \right)$

- for our case $\frac{v_1}{v_2} > 1$ (wave slows in ~~heavier~~ more massive rope)

- $\lambda_1 = \lambda_2 \left(\frac{v_1}{v_2}\right) \Rightarrow \lambda_1 > \lambda_2$

- wavelength decreases in more massive rope

- sinusoidal wave is not realistic because it must extend to infinity, but we can build any wave out of sinusoids
- long pulse close to single frequency
- very short pulse → requires broad frequency band → if wanted, must use special technique, dye as laser medium

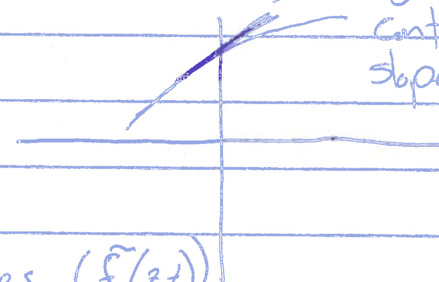
(1) $$\tilde{f}(z,t) = \begin{cases} \tilde{A}_I e^{i(k_1 z - \omega t)} + \tilde{A}_R e^{i(-k_1 z - \omega t)} & z<0 \\ \tilde{A}_T e^{i(k_2 z - \omega t)} & z>0 \end{cases}$$

- boundary conditions:

ropes are connected

$$f(0^-,t) = f(0^+,t) \rightarrow \text{continuous}$$

$$\left.\frac{\partial f}{\partial z}\right|_{0^-} = \left.\frac{\partial f}{\partial z}\right|_{0^+}$$



- true also for complex waves $(\tilde{f}(z,t))$

- applies to the real part of the wave function, but real and imaginary parts differ only by a phase shift of 90° $\cos \rightarrow \sin$, it applies to complex wave also

- $\tilde{f}(0^-,t) = \tilde{f}(0^+,t)$, $\left.\frac{\partial \tilde{f}}{\partial z}\right|_{0^-} = \left.\frac{\partial \tilde{f}}{\partial z}\right|_{0^+}$

$\Rightarrow$ plug in $z=0$ to equation (1)

$$\tilde{A}_I + \tilde{A}_R = \tilde{A}_T, \quad k_1(\tilde{A}_I - \tilde{A}_R) = k_2 \tilde{A}_T$$

$$\tilde{A}_R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)\tilde{A}_I, \quad \tilde{A}_T = \left(\frac{2k_1}{k_1 + k_2}\right)\tilde{A}_I$$

$\rightarrow$ velocities $$\tilde{A}_R = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)\tilde{A}_I, \quad \tilde{A}_T = \left(\frac{2v_2}{v_2 + v_1}\right)\tilde{A}_I$$

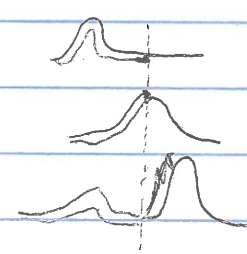
- real amplitudes + phases:

$$A_R e^{i\delta_R} = \left(\frac{v_2-v_1}{v_2+v_1}\right) A_I e^{i\delta_I}$$

$$A_T e^{i\delta_T} = \left(\frac{2v_2}{v_2+v_1}\right) A_I e^{i\delta_I}$$

- if second string is lighter ($\mu_2 < \mu_1 \Leftrightarrow v_2 > v_1$)

$\delta_I = \delta_T = \delta_R$ → all phases are the same



$$A_R = \left(\frac{v_2-v_1}{v_2+v_1}\right) A_I \; , \quad A_T = \left(\frac{2v_2}{v_2+v_1}\right) A_I$$

- $v_2 - v_1 > 0$

- second string is heavier ($v_2 < v_1$), the reflected wave is out of phase by 180°

$$(\delta_R + \pi = \delta_T = \delta_I)$$

$$\cos(-k_I z - \omega t + \delta_I - \overset{\pi}{\cancel{180}}) = -\cos(-k_I z - \omega t + \delta_I)$$

- reflected waves are upside down

$$A_R = \left(\frac{\overbrace{v_1-v_2}^{v_1-v_2>0}}{v_2+v_1}\right) A_I, \quad A_T = \left(\frac{2v_2}{v_2+v_1}\right) A_I$$

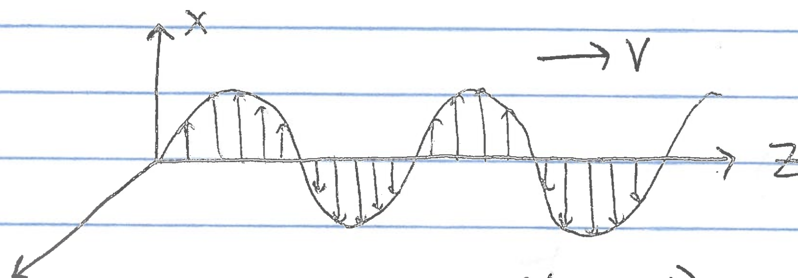
slides

-inf. massive

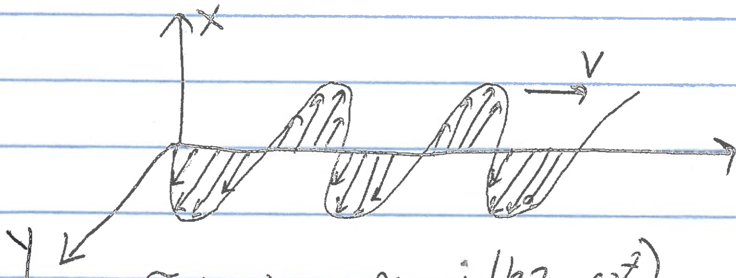
- $v_2 = 0$ $\quad A_R = A_I$ + $A_T = 0$ → no transmission (pinned to a wall) → how standing waves develop

Polarization

- transverse waves - displacement is $\perp$ to direction of energy transfer
- 2 directions are $\perp$ to this direction



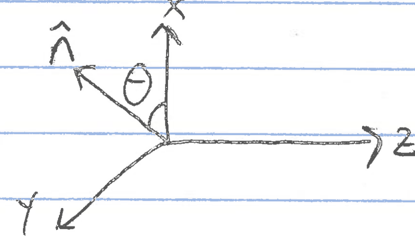
$$\tilde{f}(z,t) = \tilde{A} e^{i(kz-\omega t)} \hat{x}$$



$$\tilde{f}(z,t) = \tilde{A} e^{i(kz-\omega t)} \hat{y}$$

- or any direction in the x, y plane

$$\tilde{f}(z,t) = \tilde{A} e^{i(kz-\omega t)} \hat{n}$$



$$\hat{n} = \cos\theta \, \hat{x} + \sin\theta \, \hat{y}$$

- Superposition of two waves

$$\tilde{f}(z,t) = (\tilde{A}\cos\theta) e^{i(kz-\omega t)} \hat{x} + (\tilde{A}\sin\theta) e^{i(kz-\omega t)} \hat{y}$$

$\longrightarrow$ 2 pages

Electromagnetic Waves in a Vacuum:

- not talking about sources (accelerating charge)

- vacuum, no charge or current

$$\nabla\cdot\vec{E} = 0 \qquad \nabla\times\vec{E} = -\partial\vec{B}/\partial t$$

$$\nabla\cdot\vec{B} = 0 \qquad \nabla\times\vec{B} = \mu_0\epsilon_0\,\partial\vec{E}/\partial t$$

we can decouple $\vec{E}$ & $\vec{B}$

- $\nabla\times(\nabla\times\vec{E}) = \nabla(\cancel{\nabla\cdot\vec{E}}) - \nabla^2\vec{E} = \nabla\times\left(-\partial\vec{B}/\partial t\right)$

$= -\frac{\partial}{\partial t}(\nabla\times\vec{B})$

$\nabla\times\vec{B} = \mu_0\epsilon_0\,\partial\vec{E}/\partial t$

$$= -\frac{\partial}{\partial t}(\nabla\times\vec{B}) = -\mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$-\frac{\partial}{\partial t}\left(\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$

- $\nabla\times(\nabla\times\vec{B}) = \nabla(\cancel{\nabla\cdot\vec{B}}) - \nabla^2\vec{B} = \nabla\times\left(\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$

$$= \mu_0\epsilon_0\frac{\partial}{\partial t}(\nabla\times\vec{E}) = -\mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$\nabla^2\vec{E} \equiv (\nabla^2 E_x)\hat{x} + (\nabla^2 E_y)\hat{y} + (\nabla^2 E_z)\hat{z}$

$\nabla^2 E_x = \partial^2 E_x/\partial x^2 + \partial^2 E_x/\partial y^2 + \partial^2 E_x/\partial z^2$

$$\Rightarrow \boxed{\nabla^2\vec{E} = \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}, \quad \boxed{\nabla^2\vec{B} = \mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2}}$$

- each component satisfies:

$$\nabla^2 f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \rightarrow \text{3D wave eq.}$$

$$c = v = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8\ \text{m/s}$$

- remarkable

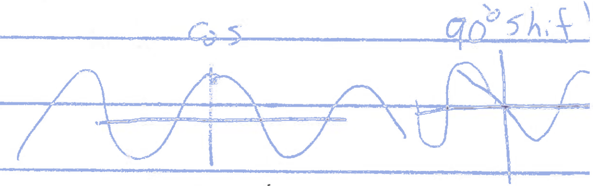
, constants in Coulomb's + Biot-Savart laws → give speed of light

+ sean +3

+ zach +3

- what if two components vertical, v, + horizontal, h, are out of phase by 90°?

$\delta_v = 0$, $\delta_h = 90°$

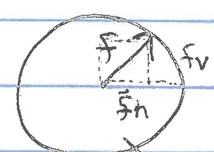


$Re(\tilde{f}_v) = \vec{f}_v = A\cos(kz - \omega t)\hat{x}$ cos shifted back 90° → $-\sin$

$Re(\tilde{f}_h) = \vec{f}_h = A \overset{\cos}{\cancel{\sin}}(kz - \omega t + 90°)\hat{y} = -A\sin(kz - \omega t)\hat{y}$

$$|\vec{f}_v|^2 + |\vec{f}_h|^2 = A^2$$

$\vec{f} = \vec{f}_v + \vec{f}_h$ is on a circle of radius A



$v \to x$

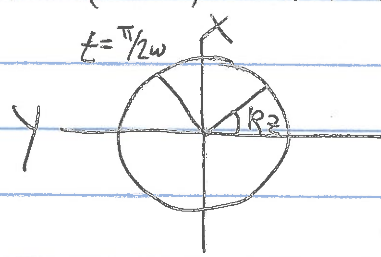
$h \to y$

@ $t = 0$, $\vec{f} = \hat{x} A\cos(kz) - A\sin(kz)\hat{y}$

$t = \pi/2\omega$, $\vec{f} = \hat{x}A\cos(kz - 90°) - A\sin(kz - 90°)\hat{y}$

↓ time at 90° @ ω

$= \cancel{A\cos}\ A\sin(kz)\hat{x} + A\cos(kz)\hat{y}$



- single frequency → monochromatic

- waves traveling in the z-direction with no x or y dependence are called "plane waves" ex.
- the fields are uniform over every plane $\perp$ to the direction of motion

$\tilde{\vec{E}}_0 = \begin{pmatrix} \tilde{E}_{0x} \\ \tilde{E}_{0y} \\ \tilde{E}_{0z} \end{pmatrix}$

$$\vec{E}(z,t) = \tilde{\vec{E}}_0 e^{i(kz-\omega t)}, \quad \vec{B}(z,t) = \tilde{\vec{B}}_0 e^{i(kz-\omega t)}$$

-Max. eq. add extra constraint

$\tilde{\vec{E}}_0$ + $\tilde{\vec{B}}_0$ are the complex amplitudes (vectors)

$\nabla \cdot \vec{E} = 0$ + $\nabla \cdot \vec{B} = 0$ (free space) $\frac{\partial \tilde{E}_{0x}}{\partial x} = \frac{\partial \tilde{E}_{0y}}{\partial y} = 0$ (const. over planes)

$$\Rightarrow (\tilde{E}_0)_z = (\tilde{B}_0)_z = 0 \qquad \nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

⇒ EM waves are transverse (functions only of z) (constant in x, y plane) "plane wave"

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{x} \ (1) = -\partial B_x/\partial t\, \hat{x}$$

$$\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{y} \ (2) = -\partial B_y/\partial t\, \hat{y}$$

$(0)\hat{z}$

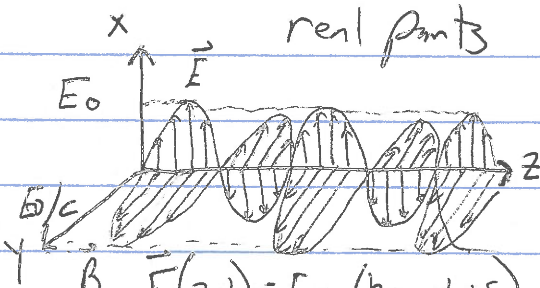
$$\nabla \times \vec{E} = -\partial \vec{B}/\partial t = (\partial B_{0x}/\partial t\, \hat{x},\ \partial B_{0y}/\partial t\, \hat{y},\ \partial B_{0z}/\partial t\, \hat{z})$$

$$\overset{(1)}{\Rightarrow} -k(\tilde{E}_0)_y = \omega(\tilde{B}_0)_x \ ; \ \overset{(2)}{k(\tilde{E}_0)_x = \omega(\tilde{B}_0)_y}$$

$$\Rightarrow \tilde{\vec{B}}_0 = \frac{k}{\omega}(\hat{z} \times \tilde{\vec{E}}_0) \text{ (amplitudes)}$$

$\vec{E}$ + $\vec{B}$ are in phase + $\perp$ to each other + the direction of prop.

real parts



-real amplitudes: $B_0 = \frac{k}{\omega} E_0 = \frac{1}{c} E_0$ ← vacuum $\boxed{\frac{k}{\omega} = c}$

$$\vec{E}(z,t) = E_0 \cos(kz - \omega t + \delta)$$
$$\vec{B}(z,t) = \frac{1}{c} E_0 \cos(kz - \omega t + \delta)$$

- Convention $\vec{E}$ is direction of polarization

$\vec{k} \equiv$ wave vector $\qquad \vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$$\tilde{\vec{E}}(\vec{r},t) = \tilde{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}\,\hat{n} \qquad (\hat{n} \to \text{polarization direction})$$

$\hat{n}\cdot\hat{k} = 0$

$$\tilde{\vec{B}}(\vec{r},t) = \frac{1}{c}\tilde{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}(\hat{k}\times\hat{n}) = \frac{1}{c}\hat{k}\times\tilde{\vec{E}}$$

* - real part:

$$\vec{E}(z,t) = E_0\cos(\vec{k}\cdot\vec{r}-\omega t+\delta)\,\hat{n}$$

$$\vec{B}(z,t) = \frac{E_0}{c}\cos(\vec{k}\cdot\vec{r}-\omega t+\delta)(\hat{k}\times\hat{n})$$

~~already did this last page~~

- energy & momentum:

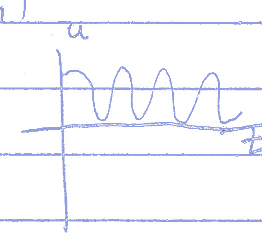
$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right), \text{ energy density}$$

- monochromatic plane wave: $\qquad c^2 = \frac{1}{\mu_0\epsilon_0}$

$$B^2 = \frac{1}{c^2}E^2 = \mu_0\epsilon_0 E^2 \Rightarrow u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\cancel{\mu_0}}\cancel{\mu_0}\epsilon_0 E^2\right)$$

→ electric & mag. contributions are equal

$$u = \epsilon_0 E^2 = \epsilon_0 E_0^2\cos^2(kz-\omega t+\delta)$$



- energy flux density (energy/area/time)

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$

- monochrome → z-direction $\qquad \vec{S} = \underbrace{c\epsilon_0 E_0^2\cos^2(kz-\omega t+\delta)\hat{z}}_{= cu\hat{z}}$

- $\vec{S}$ is energy density $\times$ wave speed

- in Δt, $l = c\Delta t$ passes through A (area)
carrying energy $\underbrace{uAc\Delta t}$ $\quad Ac\Delta t$ = volume
$u \equiv$ energy/vol

- energy/time/area $\rightarrow u \cdot c$

- EM waves also carry momentum:

$$\vec{p}_{em} = \frac{1}{c^2}\vec{S}, \quad \vec{p}_{em} \rightarrow \text{momentum density}$$

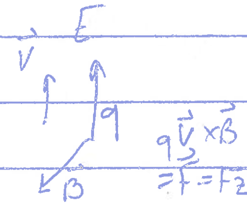
- monochrome plane wave

$$\vec{p}_{em} = \frac{1}{c}\epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)\hat{z} = \frac{1}{c}u\hat{z}$$

- for average value (cosine) $= \frac{1}{2} = \langle \cos(\omega t) \rangle$

$$\langle u \rangle = \frac{1}{2}\epsilon_0 E_0^2$$
$$\langle \vec{S} \rangle = \frac{1}{2}c\epsilon_0 E_0^2 \hat{z} \qquad \langle\,\rangle \rightarrow \text{time average}$$
$$\langle \vec{p}_{em} \rangle = \frac{1}{2}\frac{1}{c}\epsilon_0 E_0^2 \hat{z}$$

understood



393-399

- power per unit area (average)

-end Wed.

$$I \equiv \langle S \rangle = \frac{1}{2}c\epsilon_0 E_0^2$$

Monday Nov. 24

"intensity"

- perfect absorber: in Δt, mom. transf. $\Delta p = \langle \vec{p}_{em} \rangle Ac\Delta t$ volume
- pressure $P = \frac{1}{A}\frac{\Delta p}{\Delta t}^{\to F} = \frac{1}{2}\epsilon_0 E_0^2 = I/c$, perfect reflector $2\times$

Begin Monday Dec. 1 Pg. 401-405 Griffiths

EM Waves in Matter

- $\nabla \cdot \vec{D} = 0$ $\quad \nabla \times \vec{E} = -\partial \vec{B}/\partial t$
- $\nabla \cdot \vec{B} = 0$ $\quad \nabla \times \vec{H} = \partial \vec{D}/\partial t$

Maxwell's equations in matter

- linear media $\vec{D} = \epsilon \vec{E}$; $\vec{H} = \frac{1}{\mu} \vec{B}$

+ homogeneous → not varying from point to point

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \times \vec{E} = -\partial \vec{B}/\partial t$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \boxed{\mu \epsilon}\, \partial \vec{E}/\partial t$$

$\mu_0 \epsilon_0 \rightarrow \mu \epsilon$ (only variation)

$$V = \frac{1}{\sqrt{\epsilon \mu}} = \frac{c}{n}, \quad n \equiv \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}} \rightarrow \text{index of refraction}$$

- μ usually ~ 1 $\Leftarrow$ $n \sim \sqrt{\epsilon_r}$; $\frac{\mu}{\mu_0} \sim 1$

(almost all materials)

$\epsilon_r = \epsilon/\epsilon_0$

$$\overbrace{u}^{\text{energy density}} = \frac{1}{2}\left(\epsilon E^2 + \frac{1}{\mu} B^2\right), \quad \overbrace{\vec{S}}^{\text{Poynting vector}} = \frac{1}{\mu}\left(\vec{E} \times \vec{B}\right), \quad \omega = kv$$

$$|\vec{B}| = \frac{1}{v}|\vec{E}|, \quad I = \frac{1}{2}\epsilon v E_0^2$$

material 1 | material 2

boundary

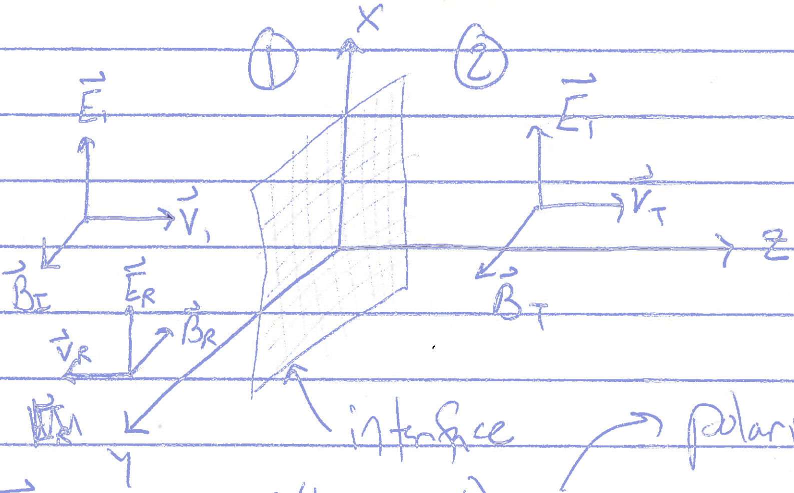
- EM boundary conditions → one material to another

(1) $\epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}$ $\qquad$ (3) $\vec{E}_1^{\parallel} = \vec{E}_2^{\parallel}$

(2) $B_1^{\perp} = B_2^{\perp}$ $\qquad$ (4) $\frac{1}{\mu_1}\vec{B}_1^{\parallel} = \frac{1}{\mu_2}\vec{B}_2^{\parallel}$

just on one side & the other

- Reflection + Transmission from normal incidence



- x, y plane is the boundary b/t 2 linear media

polarized in $\hat x$ direction

* $\tilde{\vec E}_I = \tilde E_{0I} e^{i(k_1 z - \omega t)} \hat x$

$\tilde{\vec B}_I = \tilde B_{0I} e^{i(k_1 z - \omega t)} \hat y$

Initial wave

$\tilde B_{0I} = \frac{1}{V_1} \tilde E_{0I}$

-k → to left

+k → to right

$\tilde{\vec E}_R = \tilde E_{0R} e^{i(-k_1 z - \omega t)} \hat x$

$\tilde{\vec B}_R = \tilde B_{0R} e^{i(-k_1 z - \omega t)} (-\hat y)$

Reflected wave

$\tilde B_{0R} = \frac{1}{V_1} \tilde E_{0R}$

minus sign in $\vec B$ field → $\vec B = \frac{1}{v} \hat k \times \hat E$

$\tilde{\vec E}_T(z,t) = \tilde E_{0T} e^{i(k_2 z - \omega t)} \hat x$

$\tilde{\vec B}_T(z,t) = \frac{1}{V_2} \tilde E_{0T} e^{i(k_2 z - \omega t)} \hat y$

→ k_1 in medium 1 + k_2 in medium 2

- ω stays the same (from the source)

- boundary conditions:

@ $z=0$, boundary between 2 materials

$\tilde{\vec E}_I + \tilde{\vec E}_R$ + $\tilde{\vec B}_I + \tilde{\vec B}_R$, join fields on right $\tilde{\vec E}_T + \tilde{\vec B}_T$

- there are no components $\perp$ to surface

So (1) $\epsilon_1 E_1^\perp = \epsilon_2 E_2^\perp$ } give no information
(2) $B_1^\perp = B_2^\perp$ }

*

(3) $\vec{E}_1^\parallel = \vec{E}_2^\parallel \rightarrow$ gives $\underbrace{\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}}_{\text{plug in } z=0}$

(4) $\frac{1}{\mu_1}\vec{B}_1^\parallel = \frac{1}{\mu_2}\vec{B}_2^\parallel \rightarrow$ gives $\frac{1}{\mu_1}\left(\frac{1}{v_1}\tilde{E}_{0I} - \frac{1}{v_1}\tilde{E}_{0R}\right)$
$= \frac{1}{\mu_2}\left(\frac{1}{v_2}\tilde{E}_{0T}\right)$

- $\tilde{E}_{0I} - \tilde{E}_{0R} = \beta \tilde{E}_{0T}$

letting $\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$ solving for ref. & trans in terms of initial wave

$\Rightarrow \tilde{E}_{0R} = \left(\frac{1-\beta}{1+\beta}\right)\tilde{E}_{0I}$, $\tilde{E}_{0T} = \left(\frac{2}{1+\beta}\right)\tilde{E}_{0I}$

$\rightarrow$ since $\mu_1 = \mu_2$ for most materials (or vacuum)

$\beta \approx \frac{v_1}{v_2} \Rightarrow \tilde{E}_{0R} = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)$, $\tilde{E}_{0T} = \left(\frac{2v_2}{v_2+v_1}\right)\tilde{E}_{0I}$

- in phase if $v_2 > v_1$ } same as wave on rope
- out of phase if $v_1 > v_2$ } (pg. 390)

- Real Amplitudes:

$$E_{0R} = \left(\frac{v_2 - v_1}{v_2 + v_1} \right) E_{0I} \quad , \quad E_{0T} = \left(\frac{2v_2}{v_2 + v_1} \right) E_{0T}$$

amp. positive

or in terms of indicies of refraction $n \equiv \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}}$

$$E_{0R} = \left| \frac{n_1 - n_2}{n_1 + n_2} \right| E_{0I} \; ; \; E_{0T} = \left(\frac{2n_1}{n_1 + n_2} \right) E_{0I}$$

- Intensity is defined by $I = \frac{1}{2} \epsilon v E_0^2$

- if $\mu_1 = \mu_2 = \mu_0$

- ratio of reflected intensity

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}} \right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2 \quad \text{(reflection coefficient)}$$

- transmitted

$$T \equiv \frac{I_T}{I_I} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1} \left(\frac{E_{0T}}{E_{0I}} \right)^2 = \frac{4 n_1 n_2}{(n_1 + n_2)^2} \quad \text{(transm. coefficient)}$$

$R + T = 1$ (cons. of energy requires)

Ch. 9 HW Solutions 9.1, 9.9(a), 9.10, 9.14

9.1: (for f_1 only, all others use same method)

$$f_1(z,t) = Ae^{-b(z-vt)^2}$$

* $$\frac{\partial f_1}{\partial z} = -2Ab(z-vt)e^{-b(z-vt)^2}$$

$$\frac{\partial^2 f_1}{\partial z^2} = -2Ab\left[e^{-b(z-vt)^2} - 2b(z-vt)^2 e^{-b(z-vt)^2}\right]$$

* $$\frac{\partial f_1}{\partial t} = 2Abv(z-vt)e^{-b(z-vt)^2}$$

$$\frac{\partial f_1}{\partial t} = 2Abv\left[-ve^{-b(z-vt)^2} + 2bv(z-vt)^2 e^{-b(z-vt)^2}\right] = v^2\frac{\partial^2 f_1}{\partial z^2}$$

$$\therefore \frac{\partial^2 f_1}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f_1}{\partial t^2} \checkmark$$

⋮

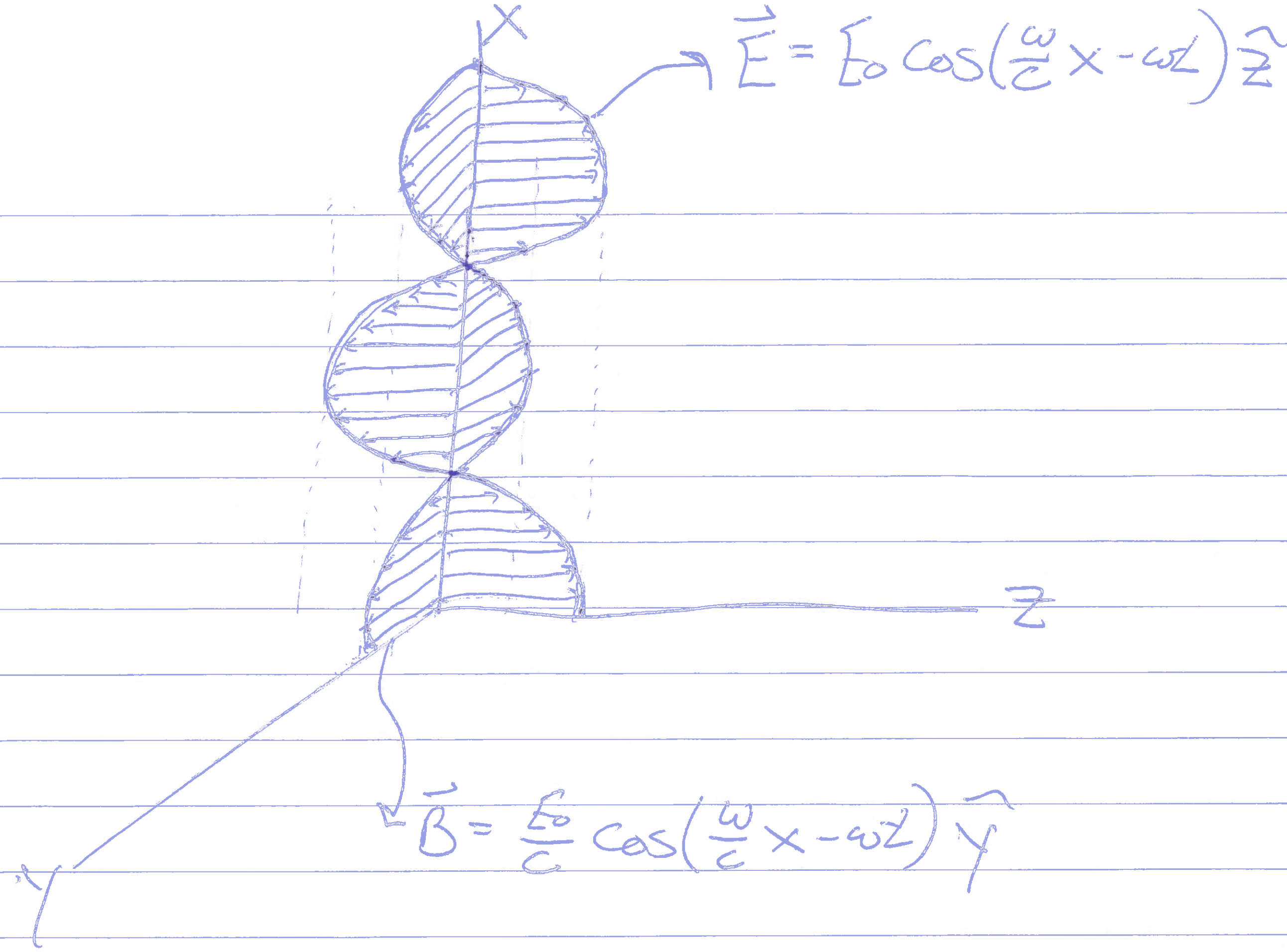
9.9 (a) → only

- Real E + B fields for monochromatic plane wave traveling in the $(-\hat{x})$ direction + polarized in the $\hat{z}$ direction

$$\left.\begin{aligned}\vec{k} &= (-\omega/c)\hat{x}\\ \hat{n} &= \hat{z}\end{aligned}\right\} \quad \vec{k}\cdot\vec{r} = (-\omega/c)\hat{x}\cdot(x\hat{x}+y\hat{y}+z\hat{z}) = (-\omega/c)x$$

$$\vec{E}(x,t) = E_0\cos\left(\frac{\omega}{c}x + \omega t\right)\hat{z}$$

$$\hat{k}\times\hat{n} = -\hat{x}\times\hat{z} = \hat{y} \qquad \vec{B}(x,t) = \frac{E_0}{c}\cos\left(\frac{\omega}{c}x + \omega t\right)\hat{y}$$



9.10: intensity of light hitting Earth

$$I = 1300\ W/m^2$$

- for a perfect absorber: $P = \frac{I}{c} = \frac{1300\ W/m^2}{3\times10^8\ m/s}$

$$\boxed{P = 4.3\times10^{-6}\ N/m^2}$$

- for a perfect reflector, power is 2x higher

$$\boxed{P = 8.6\times10^{-6}\ N/m^2}$$

- atmospheric pressure $\sim 10^5\ N/m^2 \rightarrow$ ratio $\frac{8.6\times10^{-6}}{10^5}$

$$= \boxed{8\times10^{-11}\ atm}$$

9.14 Exact reflection and transmission without assuming $\mu_1 = \mu_2 = \mu_0$.

$$R = \left(\frac{E_{0R}}{E_{0I}}\right)^2 \rightarrow \text{eq. } (9.86)$$

$$R = \left(\frac{1-\beta}{1+\beta}\right)^2, \quad \beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} \quad \text{eq. } (9.82)$$

$$T = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{E_{0T}}{E_{0I}}\right)^2 \rightarrow \text{eq. } (9.87)$$

$$T = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{2}{1+\beta}\right)^2, \quad \beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} \quad \text{eq. } (9.82)$$

$$\frac{\epsilon_2 v_2}{\epsilon_1 v_1} = \frac{\mu_1 \epsilon_2 \mu_2 v_2}{\mu_2 \epsilon_1 \mu_1 v_1} = \frac{\mu_1}{\mu_2}\left(\frac{v_1}{v_2}\right)^2 \frac{v_2}{v_1} = \frac{\mu_1 v_1}{\mu_2 v_2} = \beta$$

$$\Rightarrow T = \beta\left(\frac{2}{1+\beta}\right)^2$$

$$\therefore T + R = \frac{1}{(1+\beta)^2}\left[4\beta + (1-\beta)^2\right]$$

$$= \frac{1}{(1+\beta)^2}\left(4\beta + 1 - 2\beta + \beta^2\right) = \frac{1}{(1+\beta)^2}\left(1 + 2\beta + \beta^2\right) = 1$$

QED