Electrodynamics or Classical Electromagnetism

- large scales, for small scales QED is the correct theory
- Classical uses an extention of Newtonian mechanics

- we take it for granted that light is an E&M phenomenon, but centuries of optics passed in development w/ out this knowledge
- both E & M we likewise studied as separate entities
- E&M as 2 sides of the same coin was developed in the 19th century with the culmination occuring with special relativity
- We have talked above stationary E-fields & stationary B fields, but not how current comes about for a stationary B-field

- current density is proportional to force per unit charge

$$\vec{J} = \sigma \vec{f}, \quad \sigma \equiv \text{Conductivity} \rightarrow \frac{S^3 A^2}{kg\, m^2}$$
$$\rho \equiv 1/\sigma = \text{resistivity}$$

- Perfect conductor, $\sigma = \infty$

- we are interested in electromag. force per charge

$$\vec{J} = \sigma \underbrace{(\vec{E} + \vec{v} \times \vec{B})}_{\vec{f} = \vec{F}/q}$$

Raman #3

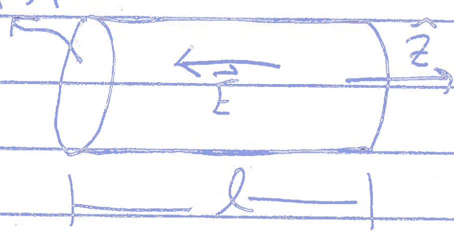
- Make the assumption that $\vec{v}$ is very small

$$\rightarrow \boxed{\vec{J} = \sigma \vec{E}} \rightarrow \text{Ohm's law}$$

Q best natural conductor (room temp.)?

- perfect conductor: $\vec{E} = \vec{J}/\sigma = 0$, inside
- for our purposes, $\sigma = \infty$

Area = A



, we have a potential across the ends, what current flows?

$$I = J \cdot A = \underbrace{\sigma E}_{J} \cdot A = \sigma A \left(\overset{E}{\frac{V}{L}}\right) = \left(\frac{\sigma A}{L}\right) V$$

$$V = I\left(\frac{L}{\sigma A}\right), \quad R = \left(\frac{L}{\sigma A}\right) \rightarrow \text{resistance (electrical)}$$

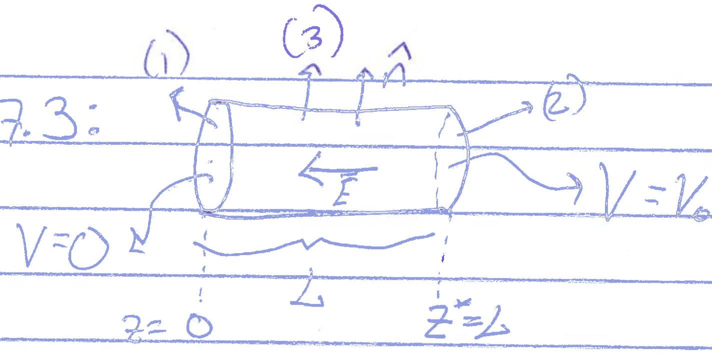
depends upon geometry + material (conductivity)

- Steady + uniform conductivity:

$$\nabla \cdot \vec{E} = \frac{1}{\sigma} \nabla \cdot \vec{J} = 0,$$ charge density = 0, charge resides on the surface (unbalanced)

- therefore, Laplace's equation holds for homogeneous, ohmic material w/ steady current

Ex 7.3:



- if V, or normal derivative of V, is specified on all surfaces, then the potential is uniquely determined

- Boundary conditions (we know pot. diff. is V_0)

$$\underset{(1)}{V(z=0)=0}, \quad \underset{(2)}{V(z=L)} = V_0 \qquad \vec{E}=-\nabla V$$

$$\rightarrow \underbrace{\vec{J}\cdot\hat{n}=0 \Rightarrow \vec{E}\cdot\hat{n}=0}_{\text{Ohm's law}} \Rightarrow \frac{\partial V}{\partial n}=0 \quad (3)$$

- Guess $V(z)=\frac{V_0 z}{L}$, $\quad V(z=0)=0 \quad (1) \checkmark$

$$V(z=L)=V_0, \quad (2) \checkmark$$

$$\frac{\partial V}{\partial n}=\frac{\partial V}{\partial s}=0 \quad (3) \checkmark$$

- $\therefore$ this must be the solution

$$\vec{E}=-\nabla V=-\frac{\partial}{\partial z}\left(\frac{V_0 z}{L}\right)=-\frac{V_0}{L}\hat{z} \quad \checkmark$$

minus sign, field points opposite to $\hat{z}$

$\vec{J}=\sigma\vec{E}$

current flows to left

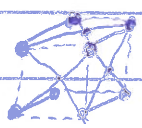
$\leftarrow (-\hat{z})$ e^- move to right

Q - What should happen to a charge according to Ohm's law?

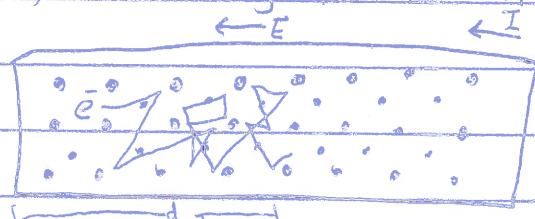
$$\vec{J} = \sigma \vec{E}$$

$$\vec{E} = \vec{F_e}/q \ , \ \vec{F} = m\vec{a}$$

- $\vec{E}$ should make charges accelerate, but $\vec{J}$ stays constant.
- individual charges are accelerated, but they scatter off other objects in material:



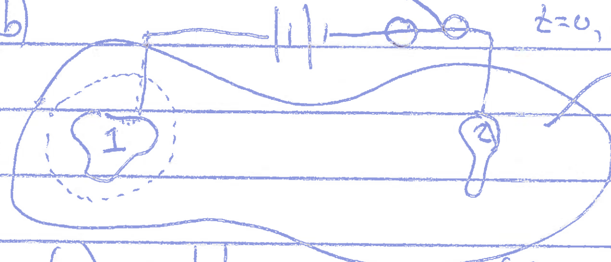
Cu (fcc)



I → Cross section of wire

- instantaneous velocity is very high
- drift velocity is low + roughly constant, $\vec{v}_{drift} = \frac{d}{t}$

(b)



$t=0$, close switch

→ Conductivity σ

(1) metal (2) metal

- resistance between (1) + (2)?

→ Gauss' law

$$I = \oint \vec{J} \cdot d\vec{a} \ , \ \vec{J} = \sigma \vec{E} \ , \ \oint \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0}$$

$$I = \sigma \oint \vec{E} \cdot d\vec{a} = \frac{\sigma Q}{\epsilon_0} \ , \ Q = CV \ , \ V = IR$$

$$I = \frac{\sigma}{\epsilon_0} CIR \Rightarrow \boxed{R = \frac{\epsilon_0}{\sigma C}}$$

@ $t=0$, open the switch, @ $t=0$, $V = V_0$ $(t=0)$

$\frac{dQ}{dt}$ is neg. losing charge

$$Q = CV = CIR \Rightarrow \frac{dQ}{dt} = -I = \frac{-Q}{RC}$$

$$\frac{dQ}{Q} = \frac{-dt}{RC} \Rightarrow \ln Q = \frac{-t}{RC} + Const$$

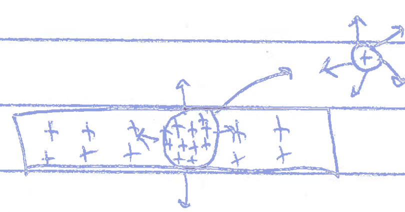
$$\rightarrow Q = Q_0 e^{t/RC}$$

$$V = \frac{Q}{C}, \quad V(t) = V_0 e^{-t/RC}, \quad \boxed{\tau = RC}$$

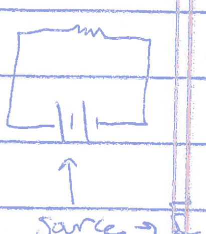
end. Wed. 22, Oct.

Electromotive force: voltage developed by a source of elec. energy (not a force, potential or energy/charge)

→ Current in a circuit is uniform



accumulation tends to make the current uniform



$$\vec{f} = \vec{f}_s + \vec{E}$$

source, such as a battery, but can be many things

- confined to one part of circuit

$\oint d\vec{l} \to$ around circuit

$$\mathcal{E} \equiv \oint \vec{f}\cdot d\vec{l} = \oint \vec{f}_s \cdot d\vec{l}, \text{ since } \underbrace{\oint \vec{E}\cdot d\vec{l} = 0}_{\text{electrostatics}}$$

$\mathcal{E} \equiv$ electromotive force, emf of the circuit

- not a force

$\vec{J} = \sigma\vec{f}$ w/ $\sigma = \infty \to$ no force needed

- ideal work source: $\vec{f} = \vec{f}_s + \vec{E} = 0 \Rightarrow \vec{E} = -\vec{f}_s$

- battery establishes potential, electrostatic field drives current

$$V = -\int_a^b \vec{E}\cdot d\vec{l} = \int_a^b \vec{f}_s \cdot d\vec{l} = \oint \vec{f}_s \cdot d\vec{l} = \mathcal{E}$$

between terminals a+b

extend to entire loop since $\vec{f}_s = 0$ there

- internal resistance of battery

$\mathcal{E} - Ir \to$ voltage between terminals

$$V = IR = \mathcal{E} - Ir$$

internal resistance

- maximum power to the load, what should R be?

$$IV = P = I^2 R, \quad I = \frac{\mathcal{E}}{r+R}$$

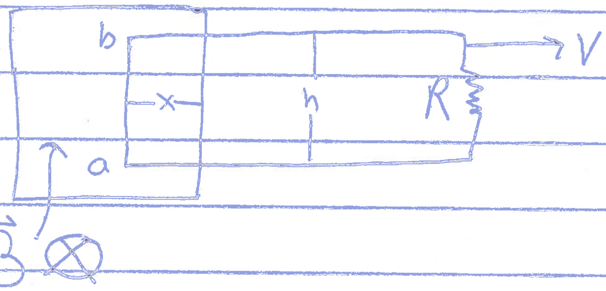
$$\Rightarrow P = \frac{\mathcal{E}^2 R}{(r+R)^2}, \quad \frac{dP}{dR} = \mathcal{E}^2\left[(r+R)^{-2} - 2R(r+R)^{-3}\right] = 0$$

$$1 - \frac{2R}{r+R} = 0 \Rightarrow \boxed{r = R}$$

Motional emf: generator → move wire in B-field

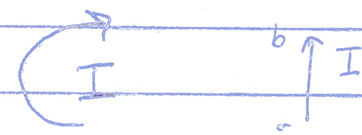
$\vec{F} = q(\vec{v} \times \vec{B})$

model:



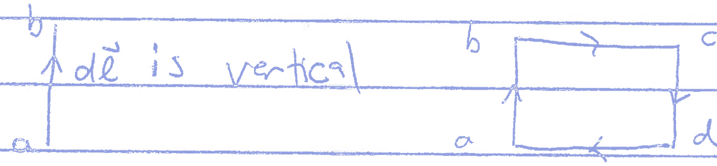
pulled to the right

- each charge in segment a–b, feels vertical force qvB



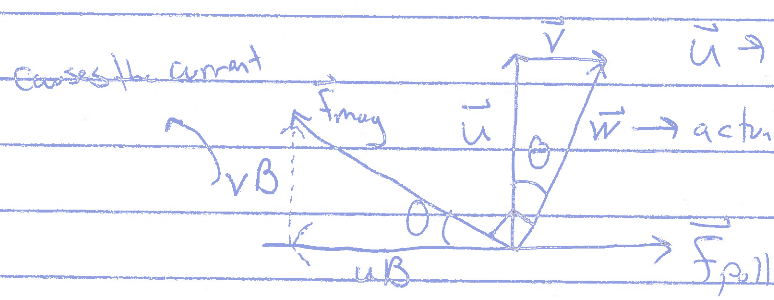
$$\mathcal{E} = \oint \vec{f}_{mag} \cdot d\vec{l} = vBh$$, no horiz. components (due to wire) — also feel vert. forces

- taken at a single instant in time (snapshot)



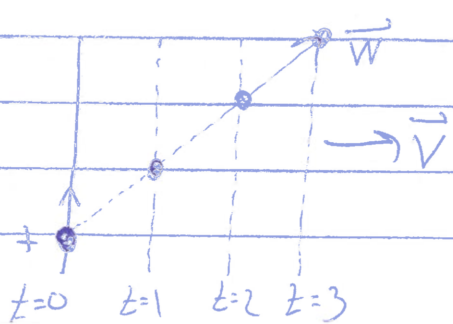
→ int. path

- mag. force establishes emf but does no work
- work done by entity pulling on loop

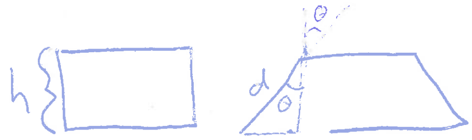


$\vec{u}$ → vert. velocity (current)

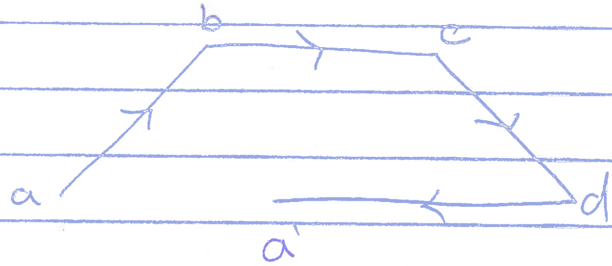
$\vec{w}$ → actual velocity

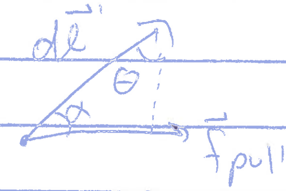


$\Rightarrow f_{pull} = uB \rightarrow \vec{u} \times \vec{B}$ → left



$\cos\theta = \frac{h}{d} \rightarrow d = \frac{h}{\cos\theta}$





$d\vec{l} \cdot \vec{f}_{pull} = dl\, f \cos\alpha$
$= dl\, f \sin\theta$

$$\frac{W}{q} = \int \vec{f}_{pull} \cdot d\vec{l} = (uB)\underbrace{\left(\frac{h}{\cos\theta}\right)}_{d}\sin\theta = vBh = \mathcal{E}$$

$d \rightarrow$ distance charge moves

$$= uBh\left(\tan\theta\right) = uBh \cdot \frac{vB}{uB} = vBh = \mathcal{E}$$

$\therefore$ all of the work is supplied by the person pulling on the loop.

- the emf generated in a moving loop:

$$\Phi \equiv \int \vec{B} \cdot d\vec{a}$$, area enclosed by single wire loop

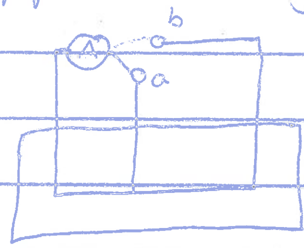
$\Phi = Bhx$ for rectangular loop

$\rightarrow$ flux changes in time $\rightarrow \frac{d\Phi}{dt} = Bh\frac{dx}{dt} = -Bhv$

$\frac{dx}{dt}$ is negative

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$, flux rule

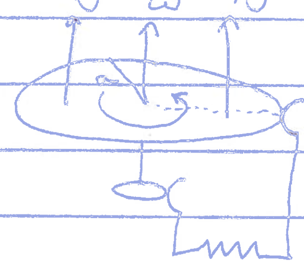
- Flux rule applies to single wire loops: paradox



- throw switch, nothing happens (no conductor moving)

- current flows due to familiar Lorentz force

Faraday Disk Ex 7.4
Faraday Dynamo



$\rightarrow$ uniform field

- radius a

$\rightarrow$ sliding contact

- speed of point on disk a distance s from center

$$v = \omega s \;,\; \vec{f}_{mag} = \vec{v} \times \vec{B} = \omega s \hat{\phi} \times B\hat{z}$$
$$= \omega s B \hat{s}$$

$$\mathcal{E} = \int_0^a \overbrace{f_{mag}\, ds}^{\vec{f}_{mag}\cdot d\vec{l}} = \omega B \int_0^a s\,ds = \tfrac{1}{2}\omega B a^2$$

$$I = \mathcal{E}/R = \frac{\omega B a^2}{2R}$$ $\Rightarrow$ "Faraday dynamo"

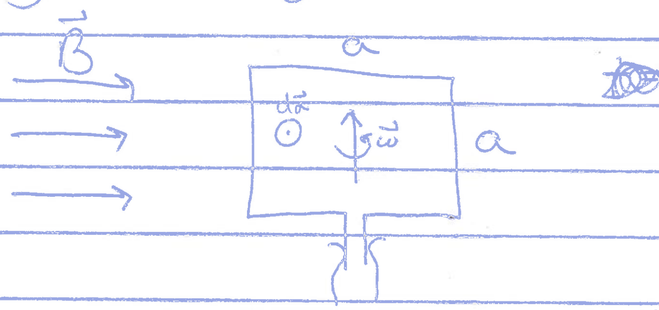
- cannot use the flux rule for this problem.

end Friday $\rightarrow$

Beg. Mon.
Oct. 27

- motional emf $\rightarrow$ $\mathcal{E} = -\dfrac{\partial \Phi}{\partial t}$

- Alternating current generator



- flux gets bigger then smaller

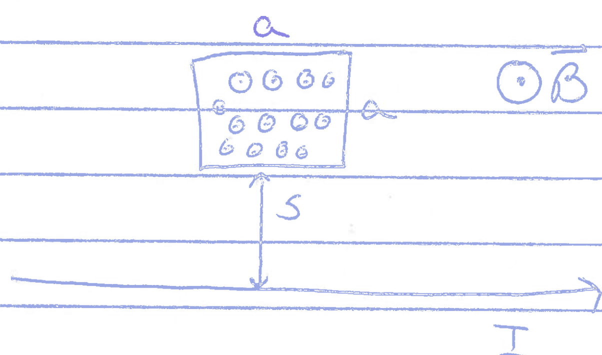
$\theta = \omega t$

$\cos\theta = \cos\omega t$

$$\Phi = \int \vec{B}\cdot d\vec{a} = \int B\,da\cos\omega t$$

$$= Ba^2\cos\omega t$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = \boxed{Ba\omega\sin(\omega t) = \mathcal{E}(t)}$$



$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

$$\Phi = \int \vec{B}\cdot d\vec{a} = \frac{\mu_0 a I}{2\pi}\int_s^{s+a}\frac{1}{s}\,ds = \frac{\mu_0 I a}{2\pi}\ln\left(\frac{s+a}{s}\right)$$

- pull w/ speed v upward

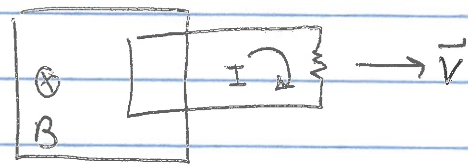
$$-\frac{d\Phi}{dt} = -\frac{\mu_0 I a}{2\pi}\frac{d}{dt}\left[\ln\left(\frac{s+a}{s}\right)\right] = -\frac{\mu_0 I a}{2\pi}\left[\frac{1}{s+a}\frac{ds}{dt} - \frac{1}{s}\frac{ds}{dt}\right] = \frac{\mu_0 I a^2 v}{2\pi s(s+a)}$$

- Counter clockwise

- Electromagnetic induction: production of an electromotive force, $\mathcal{E}$, across a conductor when exposed to an alternating B-field

- Michael Faraday: performed several experiments showing the effect, but did not have mathematical tools to describe it. → Maxwell took notice, however

(1)



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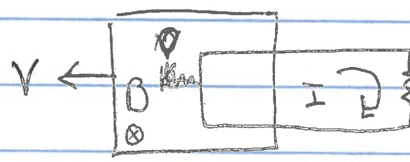

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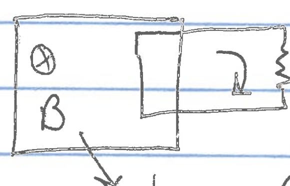


change B-field magnitude

- (1) → $\mathcal{E} = -\frac{d\Phi}{dt}$, flux rule → magnetic force responsible

(2) → the charges are not in motion, so the current cannot be ~~responsible for~~ a result of mag. force

- Idea: changing magnetic field induces an electric field

$$\Phi = \int \vec{B}\cdot d\vec{a}\ , \quad \mathcal{E} \equiv \oint \vec{f}\cdot d\vec{l} = \cancel{\oint \vec{f}_s\cdot d\vec{l}}^{\,0} + \oint \vec{E}\cdot d\vec{l}$$

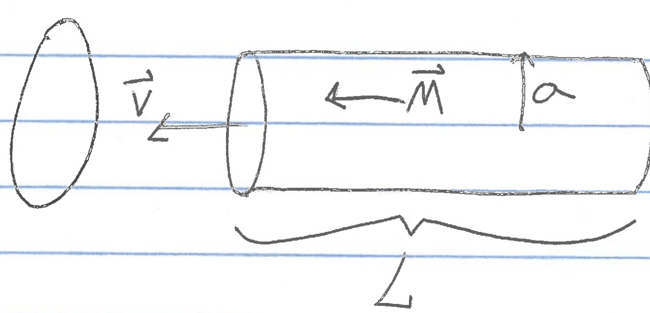
$$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt} \Rightarrow \oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{a}$$

- Stokes' theorem: $\oint \vec{E}\cdot d\vec{l} = \int (\nabla\times\vec{E})\cdot d\vec{a} = -\int\left(\frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{a}$

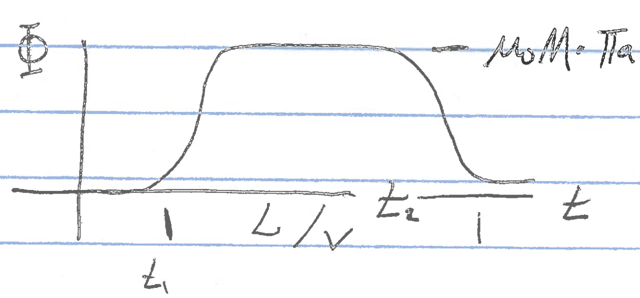
$$\Rightarrow \boxed{\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$ Faraday's law

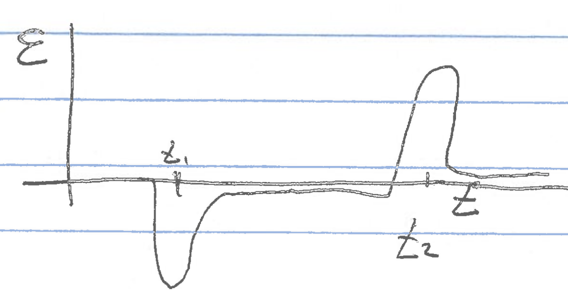
- static case $\nabla\times\vec{E}=0$, $\oint \vec{E}\cdot d\vec{l} = 0$, as expected
- constant $\vec{B}$

- the fact that for (1) + (2) produce the same exact physical effect yet there are 2 distict reasons for the enducted current lead Einstein to special relativity.



$\vec{K}_b = M\hat{\phi}$, $\vec{B} = \mu_0 \vec{M} \rightarrow$ inside, but has fringe field

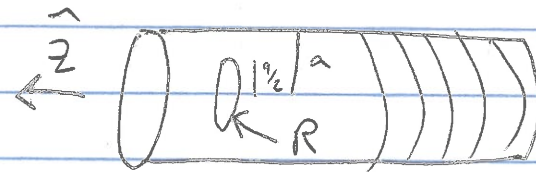




Nature abhors a change in flux

- at first, ring produces field to right → I clockwise
- as second end passes, field to left → I counterclockwise
- Lenz's law → which direction current will flow

- Solenoid:



-have them try

$$\vec{B}(t) = B_0 \cos(\omega t)\,\hat{z}$$

$$\Phi = \int \vec{B} \cdot d\vec{a} = B\pi\left(\frac{a}{2}\right)^2 = B_0\cos(\omega t)\,\pi\left(\frac{a}{2}\right)^2$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = \omega B_0 \sin(\omega t)\,\pi\left(\frac{a}{2}\right)^2$$

$$\mathcal{E} = IR \Rightarrow I = \frac{\mathcal{E}}{R} = \boxed{\frac{\pi a^2 \omega B_0 \sin(\omega t)}{4R} = I(t)}$$

- demonstration: ? Eddy currents, Neodymium, NdFeB → Boron

eddy c. - circular electric currents induced in conductors by a changing B-field → direction → Lenz's law
- EM braking
- levetation

Begin Wed. Oct. 29

- How can we exploit Faraday's law to solve for the induced E-field?

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- if E comes only from changing B,

$$\nabla \cdot \vec{E} = 0 \quad , \quad \nabla \cdot \vec{B} = 0 \quad \text{always}$$

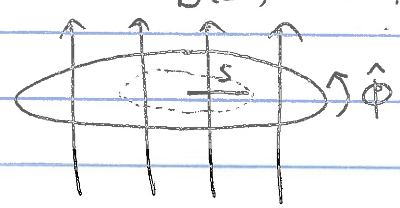
$\rho = 0$, $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\nabla \times \vec{B} = \mu_0 \vec{J}$

{ Faraday ind. E-fields are determined by $-\frac{\partial B}{\partial t}$ in same way as $\mu_0 \vec{J}$ mag. stat.

- use same tricks from Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \longrightarrow \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$$

$$\vec{B}(t) = B(t)\hat{z}$$



$$\oint \vec{E} \cdot d\vec{l} = E \cdot (2\pi s) = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\pi s^2 B(t)\right)$$

$\vec{E} = E\hat{\phi}$
$d\vec{l} = dl\,\hat{\phi}$

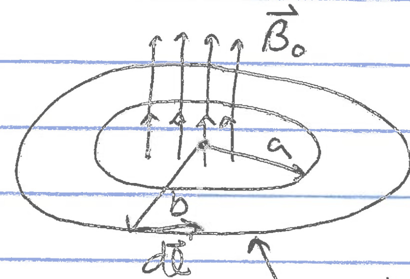
$$= -\pi s^2 \frac{dB}{dt}$$

$$\Rightarrow \vec{E} = -\frac{s}{2}\frac{dB}{dt}\hat{\phi} \rightarrow -\hat{\phi} \text{ for } +\frac{dB}{dt}$$

→ as $\vec{B}(t)$ increases, current moves clockwise (opposes changing field)

→ neg. sign indicates Lenz's law.

- wheel suspended from above, free to rotate:



$E = \hbar\omega$
$\vec{p} = \hbar\vec{k}$
$|\vec{k}| = \frac{2\pi}{\lambda}$

mom. dens.
$\vec{g} = \frac{1}{c^2}\vec{S}$
$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$
→ energy flux density
energy/area
mom/time/area

- field is turned off, what happens?

$\Phi = B \cdot \text{Area} = \vec{B}_0 \cdot \pi a^2$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt} = -\pi a^2 \frac{dB}{dt} \Rightarrow \vec{E} = \frac{-a^2}{2b}\frac{dB}{dt}\hat{\phi}$$

$E(2\pi b)$

torque on $d\vec{l} = \vec{r}\times\vec{F} = bE\lambda dl$

$E\lambda dl$
F

- torque: $\tau = b\lambda\oint \vec{E}\cdot d\vec{l} = b\lambda\left(\frac{-a^2}{2b}\frac{dB}{dt}\right)\oint dl$

$$= -b\lambda\pi a^2 \frac{dB}{dt}$$

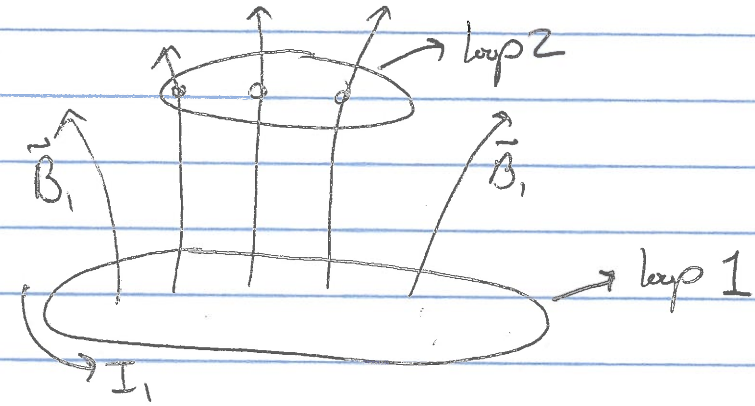
- angular momentum imparted to wheel

$\vec{\tau} = \frac{d\vec{L}}{dt} \Rightarrow$
$$L = \int \vec{\tau}\, dt = -\lambda\pi a^2 b \int_{B_0}^{0} dB = \lambda\pi a^2 b B_0$$

- independent of time → same ang. mom. applied to wheel
- conserv. of ang. mom. → means fields have momentum

*Note $\vec{L} = \vec{r}\times m\vec{v}$, $\vec{\tau} = \vec{r}\times m\vec{a}$, $\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt} = 0 + \vec{r}\times\frac{d\vec{p}}{dt} = \vec{r}\times\vec{F}$

Inductance:



- I_1 produces $\vec{B}_1$ → some lines penetrate loop 2

Φ_2 → flux of $\vec{B}_1$ through loop 2

$$\vec{B}_1 = \frac{\mu_0}{4\pi} I_1 \oint \frac{d\vec{l}_1 \times \hat{r}}{r^2} \rightarrow \text{proportional to } I_1$$

$$\Phi_2 = \int \vec{B}_1 \cdot d\vec{a}_2 = M_{21} I_1$$

$M_{21} \equiv$ mutual inductance of the two loops

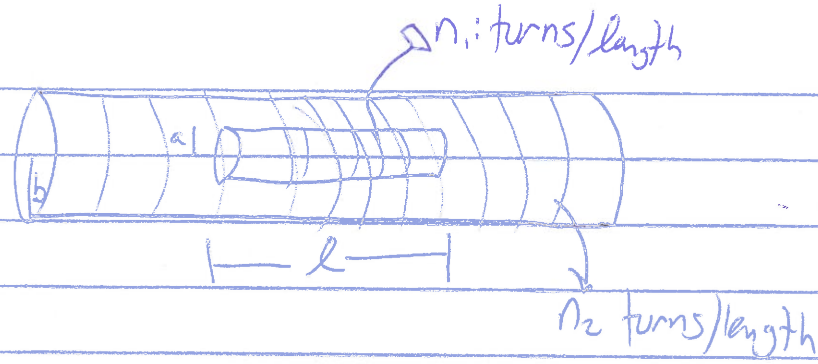
$$\Phi_2 = \int \vec{B}_1 \cdot d\vec{a}_2 = \int (\underbrace{\nabla \times \vec{A}_1}_{\text{vector pot.}}) \cdot d\vec{a}_2 = \oint \vec{A}_1 \cdot d\vec{l}_2$$

$$\vec{A}_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\vec{l}_1}{r}, \quad \Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint \left(\oint \frac{d\vec{l}_1}{r} \right) \cdot d\vec{l}_2$$

$$M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\vec{l}_1 \cdot d\vec{l}_2}{r}$$

$M_{21} = M_{12}$ → run I through loop 1 → get flux through 2

- ex. same as run I through loop 2 → flux through 1



- current I in the short solenoid, what ~~current~~ flux ~~flows~~ in long solenoid.
- short solenoid has complicated field.
- use reverse situation
- run I through outer solenoid $\rightarrow$ uniform field

$$B = \mu_0 n_2 I \rightarrow \text{long solenoid}$$

- flux through single loop of short solenoid

$$B\pi a^2 = \mu_0 n_2 I \pi a^2$$

- there are $n_1 l$ turns in short solenoid

$$\Rightarrow \quad \Phi = \mu_0 \pi a^2 n_1 n_2 l I$$

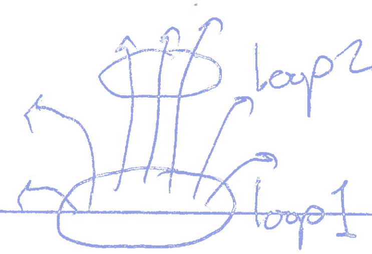
- same flux as produced by short through long

$\Phi = MI$

- mutual inductance $M = \mu_0 \pi a^2 n_1 n_2 l$

end
Wed.
Oct. 29

- Self Inductance:



EMF in wire itself

- change current in loop 1, flux through 2 changes, inducing an emf in 2 $\Rightarrow \Phi_2 = M_{21} I_1$

$$\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -M\frac{dI_1}{dt}, \quad M \equiv \text{mutual inductance}$$

- changing current also induces an emf in loop 1 as well

$$\Phi = L I, \quad L \equiv \text{self-inductance}$$

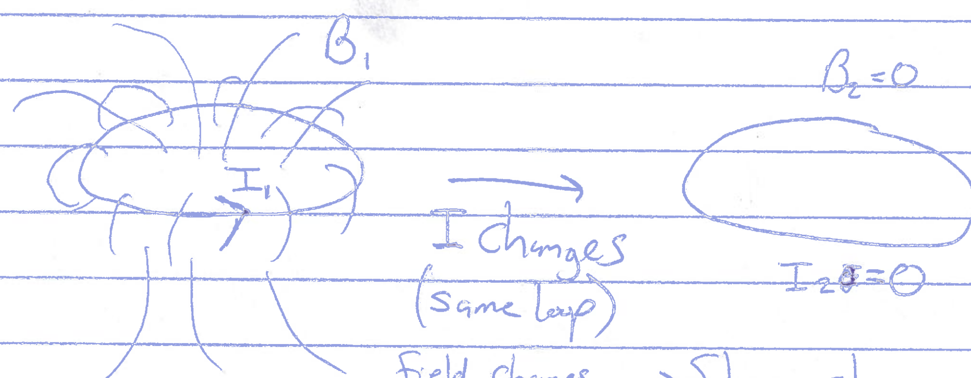
Henries (H) → V·s/A (Units)

$$\Rightarrow \mathcal{E} = -L\frac{dI}{dt} = -\frac{d\Phi}{dt}, \quad L \text{ depends on geometry.}$$

- ~~like~~ Lenz's law dictates that the emf resists changes to the current

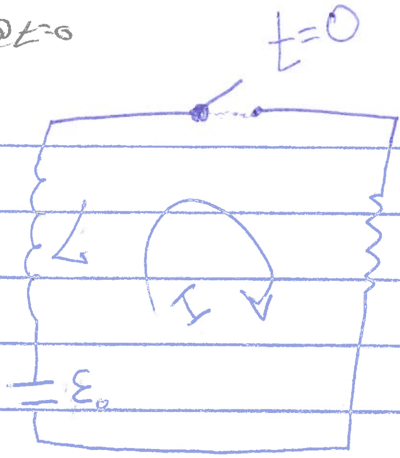
→ Inductance ⇒ back emf

Inertia — L + mass are analogues → larger L means it is more difficult to change current.



field changes → flux change → back emf arises

$I = 0$ @ $t = 0$



L = inductor

What current flows?
(when switch closed)

$\Phi = LI \qquad \frac{d\Phi}{dt} = L\frac{dI}{dt} = -\mathcal{E}$

- total emf:

$$\underbrace{\mathcal{E}_0}_{\text{emf from battery}} - \underbrace{L\frac{dI}{dt}}_{\text{back emf}} = IR$$

ASK

→ why negative; when I = const, reduces to ohms law

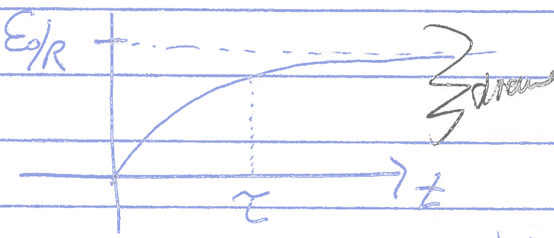
→ solution to diff. eq

$$I(t) = \underbrace{\frac{\mathcal{E}_0}{R}}_{I \text{ produced by battery}} + \underbrace{ke^{-(R/L)t}}_{I \text{ associated with Emf in ... (-) since resistive}}$$

- if at $t=0$ → $I(t) = 0$ → solve for k

$$I(t) = \frac{\mathcal{E}_0}{R}\left[1 - e^{-(R/L)t}\right]$$

- current slowly reaches $\mathcal{E}_0/R$



$\tau = L/R$ ... $\frac{\mathcal{E}_0}{R}$

Next topic

- Energy in Magnetic Fields:

- Energy is needed to drive a current (obviously) → start a current
- in particular the work against the back emf → not resistive heat
- energy is recoverable → from back emf which is lost

- work done on a unit charge for 1 loop around circuit is $-\mathcal{E}$ → work against emf
- one must supply this work to get current moving

$$\int dW = L \int I\, dI$$

→ work per unit time

$$\frac{dW}{dt} = -\mathcal{E} I = L I \frac{dI}{dt} \rightarrow \mathcal{E} = -L\frac{dI}{dt}$$

$P = IV$ Self Ind

$-\mathcal{E} = L\, dI/dt$

$\Rightarrow \boxed{W = \frac{1}{2} L I^2}$ → Start from 0 → go to final current I

(from integrating above equation

Write W another way → $\Phi = LI$ self inductance

$\vec{A} \equiv$ vector potential

→ no simple explan... serves mathematical purpose

Stokes' theorem

$$\Phi = \int \vec{B}\cdot d\vec{a} = \int (\nabla\times\vec{A})\cdot d\vec{a} = \oint \vec{A}\cdot d\vec{l}$$

$\boxed{\nabla\times\vec{A} = \vec{B}}$

→ around the circuit

$$\Phi \Rightarrow LI = \oint \vec{A}\cdot d\vec{l}$$

$$W = \frac{1}{2} I (IL) = \frac{1}{2} I \oint \vec{A}\cdot d\vec{l} = \frac{1}{2}\oint (\vec{A}\cdot \vec{I})\, d\vec{l}$$

one-dimensional case

- generalize to volume current density, $\vec{J}$

$$W = \frac{1}{2}\int (\vec{A}\cdot\vec{J})\, d\tau$$ → for volume current

3d

$\nabla\times\vec{B} = \mu_0 \vec{J}$ → Ampére's law

$$W = \frac{1}{2\mu_0}\int \vec{A}\cdot(\nabla\times\vec{B})\, d\tau$$

→ product rule #6 front of book

$$\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B})$$

$$= \vec{B}\cdot\vec{B} - \nabla\cdot(\vec{A}\times\vec{B})$$

$$W = \frac{1}{2\mu_0}\left[\int B^2 d\tau - \int \nabla\cdot(\vec{A}\times\vec{B})\, d\tau\right]$$

div. theorem

$$= \frac{1}{2\mu_0}\left[\int B^2 d\tau - \oint_S (\vec{A}\times\vec{B})\cdot d\vec{a}\right]$$

— over all volume

→ goes to zero b/c surface integral goes to zero @ ∞

$$\boxed{W_B = \frac{1}{2\mu_0}\int_{\text{all space}} B^2 d\tau}$$ → energy stored in mag. field

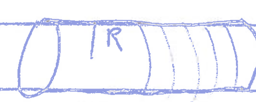
$$W_e = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 d\tau$$ → energy stored in elec. field

total energy in electromagnetic fields is

$$\boxed{u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)}$$

radius R

n → turns/length

Ex: Self-inductance of solenoid:  $B = \mu_0 n I$

7/28 - in length l there are nl turns ∴ total flux: $\Phi = \mu_0 n^2 I \pi R^2 l$

$\Phi = LI \rightarrow \boxed{\frac{L}{l} = \mu_0 n^2 \pi R^2}$ self-ind per length

- Energy stored in length l: $W = \frac{1}{2}LI^2 = \boxed{\frac{1}{2}\mu_0 n^2 \pi R^2 l I^2 = W}$

or $W = \frac{1}{2\mu_0}\int B^2 d\tau \Rightarrow \int d\tau = \pi R^2 l \rightarrow W = \frac{1}{2}\mu_0 n^2 \pi R^2 l I^2$ ✓

Maxwell's equations:

- differential form: we have discussed the following 4 equation in detail so far - mid 19^{th} century

(1) $\nabla \cdot \vec{E} = \rho/\epsilon_0$ Gauss's law
(2) $\nabla \cdot \vec{B} = 0$
(3) $\nabla \times \vec{E} = -\partial B/\partial t$ Faraday's law
(4) $\nabla \times \vec{B} = \mu_0 \vec{J}$ Ampére's law

- there is a major problem with # 4

rule: - divergence of a curl is always zero

-we have 2 curl equations

(3) $\nabla \cdot (\nabla \times \vec{E}) = \nabla \cdot \left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}(\underbrace{\nabla \cdot \vec{B}}_{0 \rightarrow \text{always}})$

div. of curl = 0 ✓ $\nabla \cdot \vec{B} = 0$ ✓

(4) $\nabla \cdot (\nabla \times \vec{B}) = \mu_0 (\nabla \cdot \vec{J})$

div. of curl = 0. $\nabla \cdot \vec{J} \neq 0$ in general

- OK for magnetostatics because $\nabla \cdot \vec{J} = 0$ ← steady currents

- for changing currents, Ampere's law is not sufficient. - needs to be amended

- the left side is most certainly zero (div. of curl always zero)

$$\nabla \cdot (\nabla \times \vec{B}) = 0$$

- so the problem is on the right: it should also equal zero.

- consider the continuity equation

$$\overbrace{\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}} = \underbrace{-\frac{\partial}{\partial t}\left(\epsilon_0 \nabla \cdot \vec{E}\right)}_{\substack{\rho = \epsilon_0 \nabla \cdot \vec{E} \\ \text{Gauss's law}}} = \underbrace{-\nabla \cdot \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)}_{\text{pull } \frac{\partial}{\partial t} \text{ into bracket}}$$

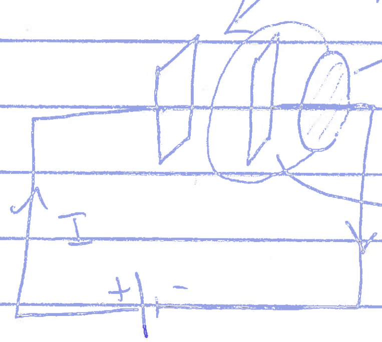
$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

$$0 = \nabla \cdot (\nabla \times \vec{B}) = \mu_0 (\vec{\nabla} \cdot \vec{J}) + \underbrace{\mu_0 \epsilon_0 \nabla \cdot \frac{\partial \vec{E}}{\partial t}}_{-\mu_0 \nabla \cdot \vec{J}} = 0$$

Maxwell "fixed" Ampère's equation

$\therefore$ a changing electric field induces a magnetic field

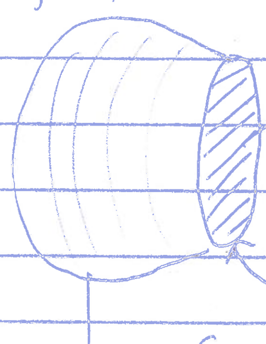
→ a second way to understand the problem:



$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$
↑ no ref. to which surface
- we use circle w/ circular

- actually → how much current pierces the surface that has the loop as the boundary
- dillemma: one surface has I going through surface, another (drawn above) has 0 current passing

-



- charge is accumulating on surface of plate → not magnetostatics → $\nabla \cdot \vec{J} \neq 0$
- resolution: assume plates are close together:

$$E = \frac{\sigma}{\epsilon_0} = \frac{1}{\epsilon_0} \frac{Q}{A} \Rightarrow \frac{\partial E}{\partial t} = \frac{1}{\epsilon_0 A} \frac{\partial Q}{\partial t} = \frac{I}{\epsilon_0 A}$$

↑ plate area

$I = \frac{dQ}{dt}$

$$\int (\nabla \times \vec{B}) \cdot d\vec{a} = \oint \vec{B} \cdot d\vec{l} = \int \mu_0 \vec{J} \cdot d\vec{a} + \mu_0 \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a}$$

→ "fixed" Ampère's law in integral form gives

$$\Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \int \left(\frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{a}$$

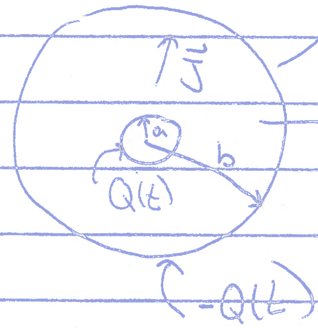
→ for surface #1 (previous page): $I_{enc} = I$, $E = 0$

→ for surface #2: $I_{enc} = 0$, $\int \underbrace{\left(\frac{\partial \vec{E}}{\partial t} \right)}_{\frac{I}{\epsilon A}} \cdot d\vec{a} = \frac{I}{\epsilon_0}$

$\Rightarrow \mu_0 \epsilon_0 \left(\frac{I}{\epsilon_0} \right) = \mu_0 I$

∴ both surfaces work.

- end Wed Nov. 5th



→ metal shells

→ conductivity σ

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Ohm's law: $\vec{J} = \sigma \vec{E} = \sigma \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$, $I = -\dot{Q} = \int \vec{J} \cdot d\vec{a} = \frac{\sigma Q}{\epsilon_0}$

$4\pi r^2$

current decreases charge

- not a static problem

$J_d \equiv$ displacement current "adds to current in Ampère's law"

$$\boxed{J_d = \epsilon_0 \frac{\partial \vec{E}}{\partial t}} = \frac{1}{4\pi} \frac{\dot{Q}}{r^2} \hat{r} = -\frac{\sigma Q}{4\pi \epsilon_0 r^2} \hat{r}$$

$\vec{J} + \vec{J}_d = 0$

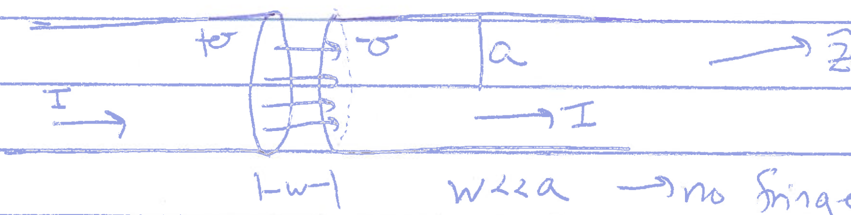
disp. current + actual current add to zero

$= -\vec{J} = -\sigma \vec{E}$ →

no magnetic field $\cancel{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}}$ $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{z} = \frac{Q}{A\epsilon_0}\hat{z}$$

B-field in the gap



$w \ll a \rightarrow$ no fringe lines

$$\vec{J}_d = \epsilon_0 \frac{\partial E}{\partial t}\hat{z} \quad \frac{dQ}{dt}\frac{1}{\pi a^2}\hat{z} = \frac{I}{\pi a^2}\hat{z}$$

- draw loop in the gap area

- Ampere's law

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 I_{d,enc}$

$s < a$

$$\oint \vec{B}\cdot d\vec{l} = B \cdot 2\pi s = \mu_0 I_{d,enc} = \frac{\mu_0 I}{\pi a^2} \cdot \pi s^2$$

$$= \mu_0 J_d \cdot \text{Area}$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 I s}{2\pi a^2}\hat{\phi}} \rightarrow \text{field in the gap.}$$

Begin Friday Nov. 7th

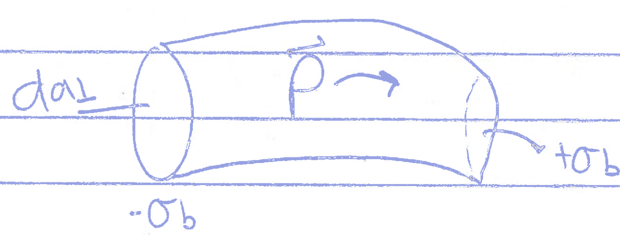
- Maxwell's equations in matter, general form, Helmholtz

- again make explicit reference to free charge + current

- static polarization: $\rho_b = -\nabla \cdot \vec{P}$

- static magnetization: $\vec{J}_b = \nabla \times \vec{M}$

- non-static case $\rightarrow$ if the polarization changes w.r.t. time we have a bound charge current $\vec{J}_p$



$\sigma_b = \vec{P}\cdot\hat{n}$

- $\vec{P}$ increases $\rightarrow$ charge on surfaces increases
- must come from a current

$$dI = \frac{\partial \sigma_b da_\perp}{\partial t} = \frac{\partial P}{\partial t} da_\perp$$

$\Rightarrow \vec{J}_p = \dfrac{\partial \vec{P}}{\partial t} \equiv$ Polarization current

- nothing to do with bound current which has to do with magnetization of object. (spin + orbital ang. mom.)

$\vec{J}_p \rightarrow$ linear motion of charge

- must satisfy cont. eq. why? → conserv. of charge

Cont. eq.

$$\nabla \cdot \vec{J} = \nabla \cdot \frac{\partial \vec{P}}{\partial t} = \frac{\partial}{\partial t}\left(\nabla \cdot \vec{P}\right) = -\frac{\partial \rho_b}{\partial t}$$

- there is no equivalent analogous accum. of charge or curr

- Charge density: $\rho = \rho_f + \rho_b = \rho_f - \nabla \cdot \vec{P}$

- current density: $\vec{J} = \vec{J}_f + \vec{J}_b + \vec{J}_p = \vec{J}_f + \nabla \times \vec{M} + \dfrac{\partial \vec{P}}{\partial t}$

- Gauss's law: $\nabla \cdot \vec{E} = \dfrac{1}{\epsilon_0}\left(\rho_f - \nabla \cdot \vec{P}\right) \Rightarrow \nabla \cdot \vec{D} = \rho_f$

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$

$\mu_0 \vec{J}$ ~~$+ \mu_0\epsilon_0 \partial\vec{E}/\partial t$~~

- Ampere's law: $\nabla \times \vec{B} = \mu_0\left(\vec{J}_f + \nabla \times \vec{M} + \partial \vec{P}/\partial t\right) + \mu_0\epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}, \quad \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

$\dfrac{\partial \vec{D}}{\partial t} \rightarrow$ "displacement current"

- for free charges & currents:

$$\boxed{\begin{array}{ll} \nabla \cdot \vec{D} = \rho_f & \nabla \times \vec{E} = -\partial \vec{B}/\partial t \\ \nabla \cdot \vec{B} = 0 & \nabla \times \vec{H} = \vec{J}_f + \partial \vec{D}/\partial t \end{array}}$$

end Friday Nov. 17

- linear media:

$\vec{P} = \epsilon_0 \chi_e \vec{E}$ $\qquad \vec{M} = \chi_m \vec{H}$

$\vec{D} = \epsilon \vec{E}$ $\qquad \vec{H} = \frac{1}{\mu}\vec{B}$

$\epsilon \equiv \epsilon_0(1+\chi_e)$, $\mu \equiv \mu_0(1+\chi_m)$

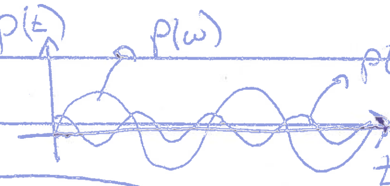
- non-linear optics:

$P(t) = \epsilon_0\left(\chi^{(1)}\vec{E}(t) + \chi^{(2)}\vec{E}^2(t) + \ldots\right)$

$\chi^{(n)} \rightarrow n^{th}$ order sus.

- Applied field: $E_0 e^{-i\omega t}$

$\Rightarrow P(t) = \epsilon_0\left(\chi^{(1)}E_0 e^{-i\omega t} + \chi^{(2)}E_0^2 e^{-2i\omega t} + \ldots\right)$



- General Boundary Conditions for E & M:

Maxwell's equations:

(1) $\oint_S \vec{D}\cdot d\vec{a} = Q_{f\,enc}$

(2) $\oint_S \vec{B}\cdot d\vec{a} = 0$

$\Big\}$ Closed surfaces

$\nabla\cdot\vec{D} = \rho_f$

$\nabla\cdot\vec{B} = 0$

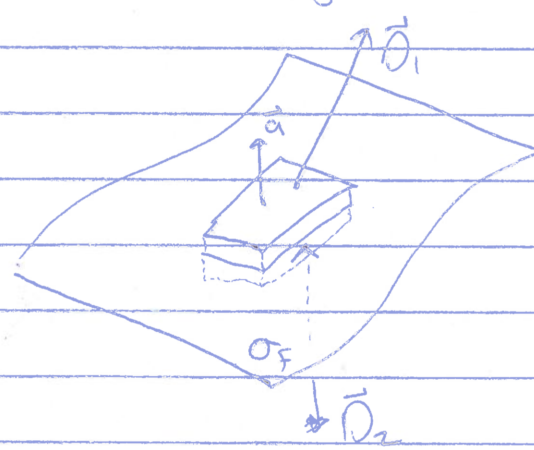
$\nabla\times\vec{E} = -\partial\vec{B}/\partial t$

$\nabla\times\vec{H} = \vec{J}_f + \partial\vec{D}/\partial t$

(3) $\oint_P \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{a}$

(4) $\oint_P \vec{H}\cdot d\vec{l} = I_{f,enc} + \frac{d}{dt}\int_S \vec{D}\cdot d\vec{a}$

- from (1)



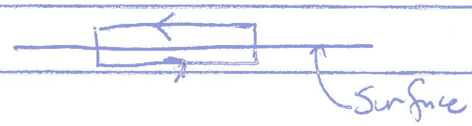
$\vec{D}_1\cdot\vec{a} - \vec{D}_2\cdot\vec{a} = \sigma_f a$

$\perp$ to surface

$\boxed{D_1^{\perp} - D_2^{\perp} = \sigma_f}$

From (2) $\longrightarrow$ $B_1^{\perp} - B_2^{\perp} = 0$ (continuous across boundary)

- very thin Amperian loop $\rightarrow$ (3)



$$\vec{E}_1 \cdot d\vec{l} - \vec{E}_2 \cdot \vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$$

$\rightarrow$ flux $\rightarrow 0$ as thickness of loop $\rightarrow 0$

$\Rightarrow \vec{E}_1^{\parallel} - \vec{E}_2^{\parallel} = 0$, two ends also go to 0

· parallel components of E-field are continuous

(4) $\rightarrow$ $\vec{H}_1 \cdot \vec{l} - \vec{H}_2 \cdot \vec{l} = I_{f\,enc}$

$\hat{n} \times \vec{l}$ → normal to loop

$I_{f\,enc} = \vec{K}_f \cdot (\hat{n} \times \vec{l}) = (\vec{K}_f \times \hat{n}) \cdot \vec{l} = \vec{H}_1 \cdot \vec{l} - \vec{H}_2 \cdot \vec{l}$

$\Rightarrow \vec{H}_1^{\parallel} - \vec{H}_2^{\parallel} = \vec{K}_f \times \hat{n}$

- for linear media: $\vec{P} = \epsilon_0 \chi_e \vec{E}$; $\vec{M} = \chi_m \vec{H}$; $\vec{D} = \epsilon \vec{E}$; $\vec{H} = \frac{1}{\mu}\vec{B}$

- no charge/no current (free)

(1) $\epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma_f$	$\epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = 0$
(2) $B_1^{\perp} - B_2^{\perp} = 0$	$B_1^{\perp} - B_2^{\perp} = 0$
(3) $\vec{E}_1^{\parallel} - \vec{E}_2^{\parallel} = 0$	$\vec{E}_1^{\parallel} - \vec{E}_2^{\parallel} = 0$
(4) $\frac{1}{\mu_1}\vec{B}_1^{\parallel} - \frac{1}{\mu_2}\vec{B}_2^{\parallel} = \vec{K}_f \times \hat{n}$	$\frac{1}{\mu_1}\vec{B}_1^{\parallel} - \frac{1}{\mu_2}\vec{B}_2^{\parallel} = 0$

vector b/c parallel has 2 components

7.44: Perfect Conductor

$\vec{J} = \sigma \vec{E} \Rightarrow \vec{E} = 0 \longrightarrow$ net charge will only be present on the surface

(a) Show that mag. field is constant inside

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{E} = 0 \rightarrow \frac{\partial \vec{B}}{\partial t} = 0 \rightarrow \vec{B}(\vec{r})$$

(b) mag. flux through a loop of perf. cond. is const

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt} = 0 \rightarrow \Phi \text{ const. in time}$$

(c) Current in supercond. is on surface:

assumed

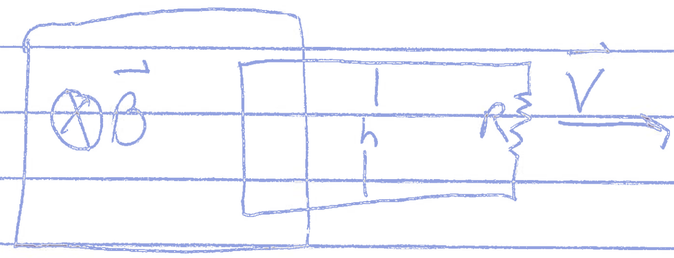
$* \vec{B} = 0 \Rightarrow$ perfect diamagnetism

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$\vec{E} = 0$, $\vec{B} = 0 \Rightarrow \vec{J} = 0 \rightarrow$ current on surface only

(d) X

7.55:



$$\mathcal{E} = vBh = IR \rightarrow I = \frac{vBh}{R}$$

- what if the wire has no resistance?
- current is then limited only by back emf from self inductance L (normally small)

$$\mathcal{E} = vBh = -L\frac{dI}{dt} \; ; \; F = IhB \rightarrow \text{Lorentz}$$

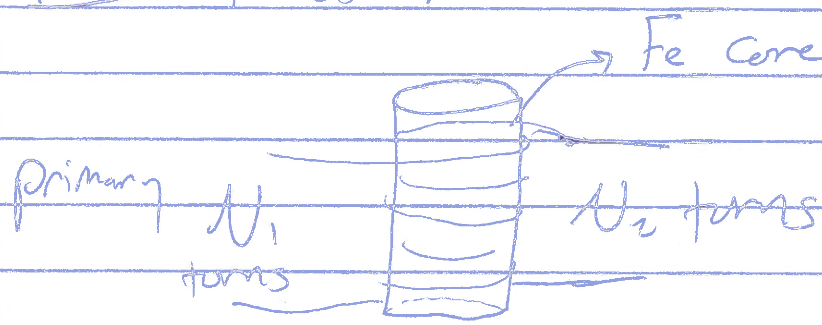
$$IhB = ma = m\frac{dv}{dt} \Rightarrow m\frac{d^2v}{dt^2} = hB\frac{dI}{dt}$$

$$\Rightarrow \frac{d^2v}{dt^2} = +\frac{hB}{m}\underbrace{\left(\frac{-hB}{L}\right)v}_{dI/dt} \Rightarrow \frac{d^2v}{dt^2} = -\omega^2 v$$

$$\boxed{\omega = \frac{hB}{\sqrt{mL}}}$$

- simple harmonic oscillator
- that is the loop of mass m

7.57: Transformer



- current I in primary is changing:

$\Phi \rightarrow$ flux through a single loop

$$\Phi_1 = N_1 \Phi$$

$$\Phi_2 = N_2 \Phi$$

$$\mathcal{E}_1 = -N_1 \frac{dI}{dt} \; ; \quad \mathcal{E}_2 = -N_2 \frac{dI}{dt}$$

$$\Rightarrow \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$$

- start with 1000 V, we want 10,000 V

if we ~~want~~ have 10 turns for primary → wrap second coil 100 times

- 120 V out of wall → 240 by winding 2x as many turns to output
- P = IV → current reduced

Conservation in E+M

- Energy + Momentum (linear + angular) cannot change in a closed system, total amount → can change form
  - in E+M energy transfered between fields + particles
  - momentum is also conserved (fields carry momentum)

- conservation of charge: charge can be neither created nor destroyed + charge - neg charge is conserved.

- locally: for a volume amount of charge is equal to the amount of charge flowing in - charge flowing out
- continuity equation:

$$Q(t_2) = Q(t_1) + Q_{in} - Q_{out}$$

- individual charges can be destroyed or created, but equal amounts of + + - are created/destroyed simultaneously.

$$Q(t) = \int_{vol} \rho(\vec{r}, t)\, d\tau \rightarrow \text{total charge in volume}$$

- current flowing through volume's boundary

$$I = \frac{dQ}{dt} = -\int_S \vec{J} \cdot d\vec{a} \Rightarrow \int_{vol} \frac{\partial \rho}{\partial t} d\tau = -\int_{vol} \nabla \cdot \vec{J}\, d\tau$$

$$\boxed{\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{J}}$$ → not independent assumption → derived from Max. eqn.

$$W_e = \frac{\epsilon_0}{2}\int E^2 \, d\tau$$ → work to assemble static charge dist.

$$W_m = \frac{1}{2\mu_0}\int B^2 \, d\tau$$ → work to move charges against back emf

stored energy per unit volume: $u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$

- energy conservation:

  - have charges + currents producing E + B

  - in dt, how much work is done on charges (dW)?

$$dW = \vec{F}\cdot d\vec{l} = q(\vec{E} + \vec{v}\times\vec{B})\cdot\vec{v}\,dt = q\vec{E}\cdot\vec{v}\,dt$$

Volume: $q \rightarrow \rho\, d\tau$, $\rho\vec{v} = \vec{J} \Rightarrow$ $\rho\, d\tau\, \vec{E}\cdot\vec{v}\,dt = \vec{E}\cdot(\rho\vec{v})\,d\tau\, dt$

$$\frac{dW}{dt} = \int(\vec{E}\cdot\vec{J})\,d\tau$$

$$\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} \Rightarrow \vec{J} = \frac{1}{\mu_0}\left(\vec{E}\cdot(\nabla\times\vec{B}) - \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{E}\cdot\vec{J} = \frac{1}{\mu_0}\vec{E}\cdot(\nabla\times\vec{B}) - \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

#6 $$\nabla\cdot(\vec{E}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot(\nabla\times\vec{B})$$

$\nabla\times\vec{E} = -\partial\vec{B}/\partial t$ – Faraday's law

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} = B_x \frac{\partial B_x}{\partial t} + B_y \frac{\partial B_y}{\partial t}$$

$$\frac{1}{2}\frac{\partial}{\partial t} B^2 = \frac{1}{2}\left(2B_x \frac{\partial B_x}{\partial t} + 2B_y \frac{\partial B_y}{\partial t} + \dots\right.$$

$$\Rightarrow \vec{E} \cdot (\nabla \times \vec{B}) = -\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} - \nabla \cdot (\vec{E} \times \vec{B})$$

$$\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t} B^2, \quad \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t} E^2$$

$$\Rightarrow \vec{E} \cdot \vec{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right) - \frac{1}{\mu_0} \nabla \cdot (\vec{E} \times \vec{B})$$

div. theorem

$$\Rightarrow \frac{dW}{dt} = -\frac{d}{dt}\int \underbrace{\frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)}_{u_{em}} d\tau - \frac{1}{\mu_0} \oint_S (\vec{E} \times \vec{B}) \cdot d\vec{a}$$

in volume — flowing outside volume

"Poynting's theorem"

- work done on charges = − energy in fields − energy that flowed out through the surface.

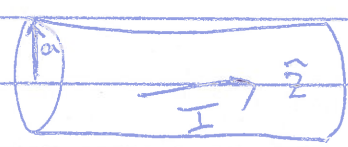
(in volume)

$$\vec{S} \equiv \frac{1}{\mu_0} \vec{E} \times \vec{B} \rightarrow W/m^2$$

"energy flux density"

* most consider energy of both fields + particles

$$\frac{dW}{dt} = -\frac{d}{dt}\int u_{em}\, d\tau - \oint \vec{S} \cdot d\vec{a}$$



- energy per unit time delivered to wire?

$$\vec{E} = \frac{V}{L}\hat{z} \qquad \vec{B} = \frac{\mu_0 I}{2\pi a}\hat{\phi}$$

s, ϕ, z

Poynting vector:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{1}{\mu_0}\frac{V}{L}\frac{\mu_0 I}{2\pi a}(-\hat{s})$$

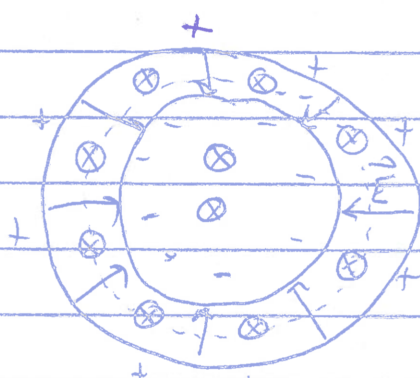
- radially inward

- energy per unit time passing in through the surface

$$\int \vec{S}\cdot d\vec{a} = \frac{1}{\mu_0}\frac{V}{L}\frac{\mu_0 I}{2\pi a}\underbrace{\int d\vec{a}}_{2\pi a L}$$

$$= V\cdot I = \text{Power} = \frac{dW}{dt}$$

- will come into play for E&M waves
- E&M fields also have momentum $\vec{p} = \mu_0\epsilon_0\int \vec{S}\,d\tau$
- angular momentum also conserved



- capacitor

- $\vec{E}$ - radially in
- $\vec{B}$ - into page
- $\vec{S} \to$ circum. $\hat{\phi}$

- momentum density proportional to energy flow density
- static field → flowing momentum
- when discharges, E-field drops, B-constant → momentum drops
- Lorentz force in direction of momentum → current gains this mom, → fields loose it

- Current is charge moving   Exam II Review Nov. 19th 2014
- to move charge (to get it moving) on needs a force

$\vec{J} = \sigma \vec{f}$, $\vec{f} \equiv$ force per unit charge

$\sigma \equiv$ conductivity

$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$, assume $\vec{v}$ is low

$\boxed{\vec{J} = \sigma \vec{E}}$ Ohm's law $\rightarrow$ common $V = IR$

- perfect conductor $\underline{\sigma = \infty}$
- $R \equiv$ resistance $\rightarrow$ geometrical + material property
- for steady currents $\nabla \cdot \vec{E} = \frac{1}{\sigma} \nabla \cdot \vec{J} = 0$
- residual charge is on the surface $\rightarrow \rho = 0$
- $\vec{J}$ is uniform in a conductor due to interactions with the material $\rightarrow$ other electrons + ions in metal
  - predicted to accelerate
- Power delivered to the wire $P = IV = I^2 R$ (friction)
- Electromotive Force, $\mathcal{E}$ (really a potential)

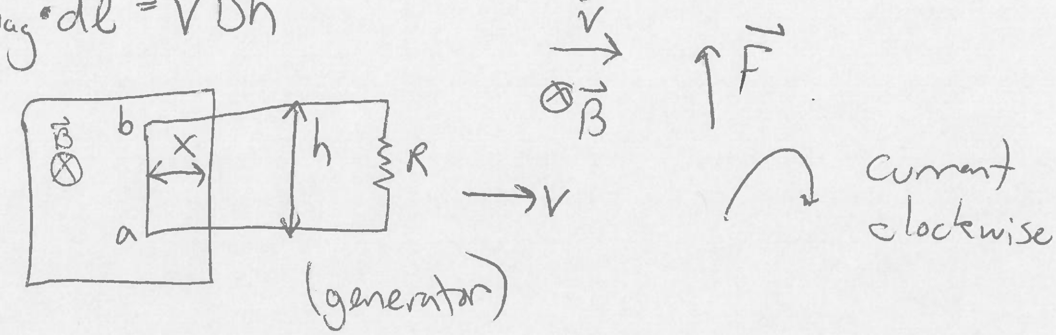
$$\mathcal{E} = \oint \vec{f} \cdot d\vec{l} = \oint \vec{f}_s \cdot d\vec{l} \text{ , for circuit}$$

- ideal source $\vec{E} = -\vec{f}_s$ $\rightarrow$ minus sign $\rightarrow$ conventional

$$V = -\int_a^b \vec{E} \cdot d\vec{l} = \int_a^b \vec{f}_s \cdot d\vec{l} = \oint \vec{f}_s \cdot d\vec{l} = \mathcal{E}$$

- Motional emf → move a wire through a B-field

$$\mathcal{E} = \oint \vec{f}_{mag} \cdot d\vec{l} = vBh$$

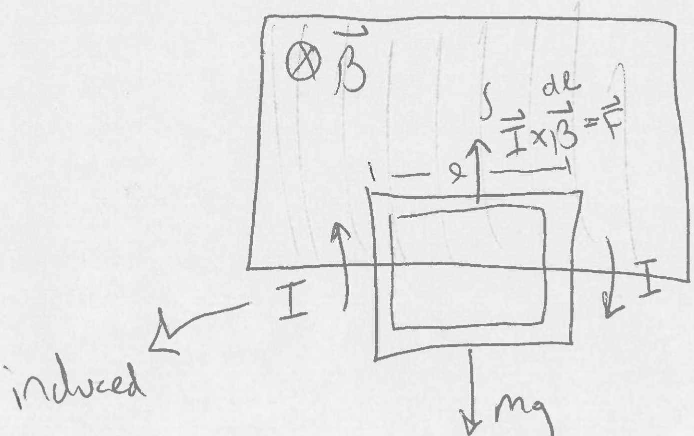


- $\Phi \equiv \int \vec{B} \cdot d\vec{a}$, flux

$$\Phi = Bhx, \quad \frac{d\Phi}{dt} = Bh\frac{dx}{dt} = -Bhv$$

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$ – flux rule for emf

- eddy currents lead to electromagnetic braking
  - move magnet near a conductor → feel a drag force



$$\mathcal{E} = Blv = IR$$

$$I = \frac{Bl}{R} v$$

upward B-force

$$IlB = \frac{B^2 l^2}{R} v$$

$$mg - \frac{B^2 l^2}{R} v = m\frac{dv}{dt} \; ; \; \frac{dv}{dt} = g - \alpha v, \quad \alpha = \frac{B^2 l^2}{mR}$$

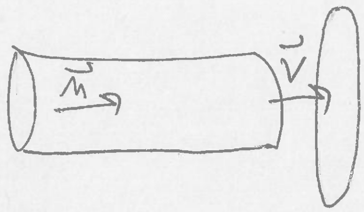
$g = \alpha v_t$, terminal velocity → $\boxed{v_t = \frac{g}{\alpha} = \frac{mgR}{B^2 l^2}}$

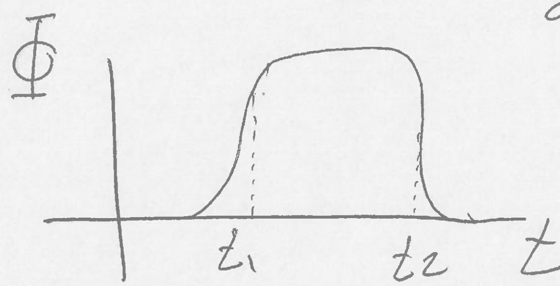
$\Sigma F = 0, \; \frac{dv}{dt} = 0$

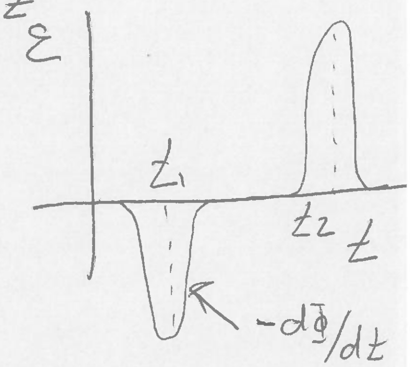
- A changing magnetic field induces an electric field

  - Faraday's law of induction

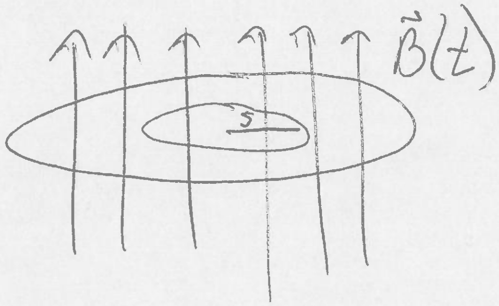
$$\nabla \times \vec{E} = -\partial \vec{B}/\partial t \quad , \quad \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi}{dt}$$







- minus sign in flux rule (is Lenz's law)

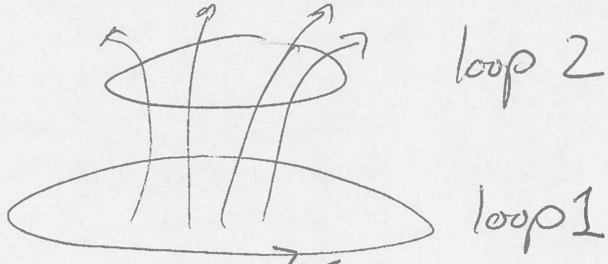
-


$$\oint \vec{E} \cdot d\vec{\ell} = E(2\pi s) = -\frac{d\Phi}{dt}$$

$$= -\frac{d}{dt}\left(\pi s^2 B(t)\right) = -\pi s^2 \frac{dB}{dt}$$

$$\Rightarrow \boxed{\vec{E} = -\frac{s}{2}\frac{dB}{dt}\hat{\phi}}$$

- <u>Inductance</u>:



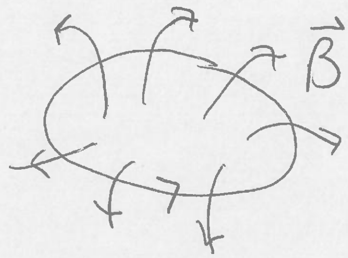
$$\Phi_2 = M_{21} I_1 \Rightarrow \Phi_1 = M_{12} I_2$$

$$M_{21} = M_{12}$$

$$\boxed{\Phi = M I} \rightarrow \text{mutual inductance}$$

- self-inductance

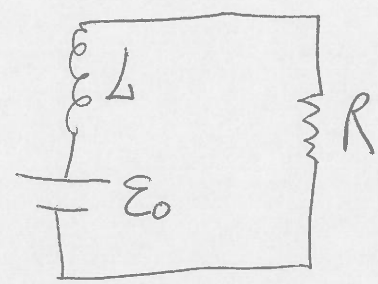
- wire loop produces a flux through itself



$$\Phi = L I$$

self-inductance

$$\mathcal{E} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

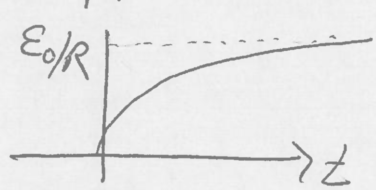


$$\mathcal{E}_0 - L\frac{dI}{dt} = IR$$

$$I(t) = \frac{\mathcal{E}_0}{R} + ke^{-(R/L)t}, \quad I(t) = 0,\ t = 0$$

$$k = -\frac{\mathcal{E}_0}{R}$$

$$I(t) = \frac{\mathcal{E}_0}{R}\left[1 - e^{-(R/L)t}\right]$$



- energy needed to start current against back emf from circuit's inductance → energy is stored, and one can get this back.

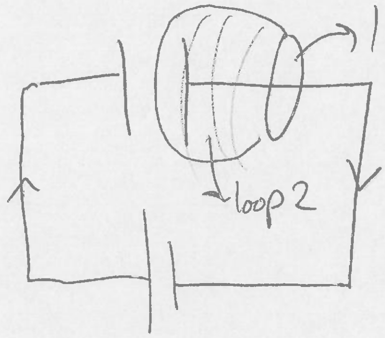
Maxwell's equations

$$\nabla \cdot \vec{E} = \rho/\epsilon_0 \qquad \nabla \times \vec{E} = -\partial\vec{B}/\partial t$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \, \partial\vec{E}/\partial t$$

} Know these!

- displacement current: $\vec{J}_d \equiv \epsilon_0 \, \partial\vec{E}/\partial t$, $I_d = \int \vec{J}_d \cdot d\vec{a}$



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi s} \hat{\phi}}$$

$$|\vec{E}| = \frac{Q}{A}, \quad \frac{\partial E}{\partial t} = \frac{I}{A}$$

$$\oint \vec{B} \cdot d\vec{l} = \underbrace{\cancel{\mu_0 I_{enc}}}_{0} + \mu_0 \epsilon_0 \int \left(\frac{\partial \vec{E}}{\partial t}\right) \cdot d\vec{a}$$

$$\mu_0 \epsilon_0 \int \left(\partial\vec{E}/\partial t\right) \cdot d\vec{a} = \mu_0 I$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi s} \hat{\phi}}$$

- in matter $\vec{J}_p = \partial\vec{P}/\partial t \rightarrow$ third current

$$\vec{J} = \vec{J}_f + \vec{J}_b + \vec{J}_p = \vec{J}_f + \nabla \times \vec{M} + \partial\vec{P}/\partial t$$

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J}_f + \nabla \times \vec{M} + \partial\vec{P}/\partial t\right) + \mu_0 \epsilon_0 \, \partial\vec{E}/\partial t$$

$$\nabla \times \vec{H} = \vec{J}_f + \partial\vec{D}/\partial t \; ; \quad \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

$$\nabla \cdot \vec{D} = \rho_f \; ; \quad \vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\frac{\partial \vec{D}}{\partial t} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t}$$