Chapter 6:

Magnetic Fields in Matter

- mag. in matter is caused by electron orbits and spin

- So small, they can be considered dipoles
- normally, the dipoles cancel due to random alignment of dipoles
- when field is applied, dipoles align & medium becomes magnetically polarized or magnetized.

- parallel to B $\longrightarrow$ paramagnetic

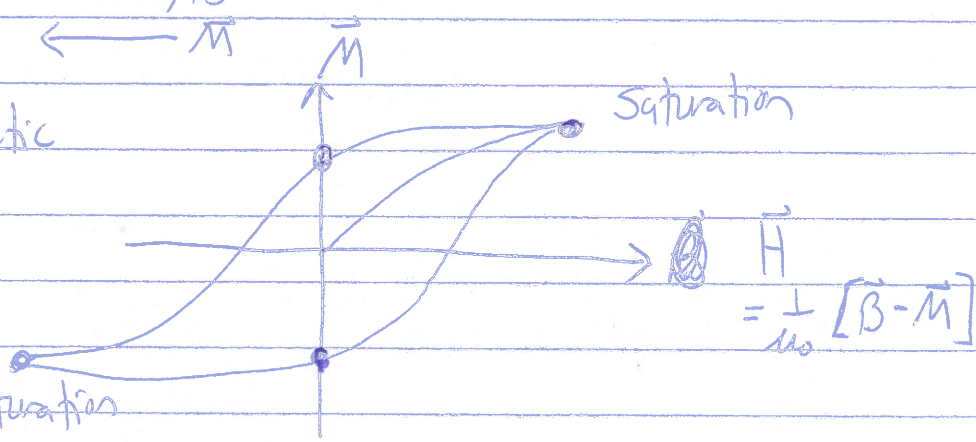
$$\longrightarrow \vec{B}$$
$$\longrightarrow \vec{M}$$

- anti-parallel $\longrightarrow$ diamagnetic

$$\longrightarrow \vec{B}$$
$$\longleftarrow \vec{M}$$

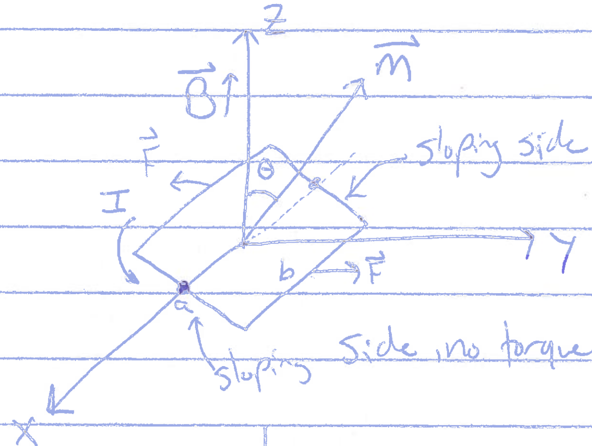
- Ferromagnetic

- non-linear



- Square loop, tilted from z-axis toward y-axis

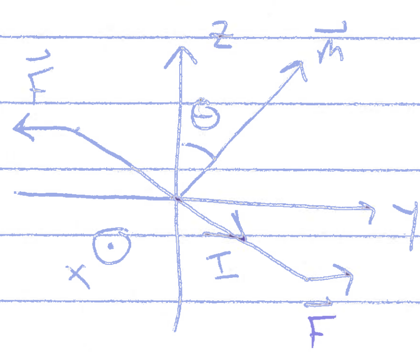
- $\vec{B}$ - in z-direction



$\vec{N}_L = \frac{a}{2} F \sin\theta \hat{x}$

$\vec{N}_R = \frac{a}{2} F \sin\theta \hat{x}$

$\vec{N}_L + \vec{N}_R = \vec{N} = aF\sin\theta\,\hat{x}$



$F = IbB \rightarrow$ Lorentz

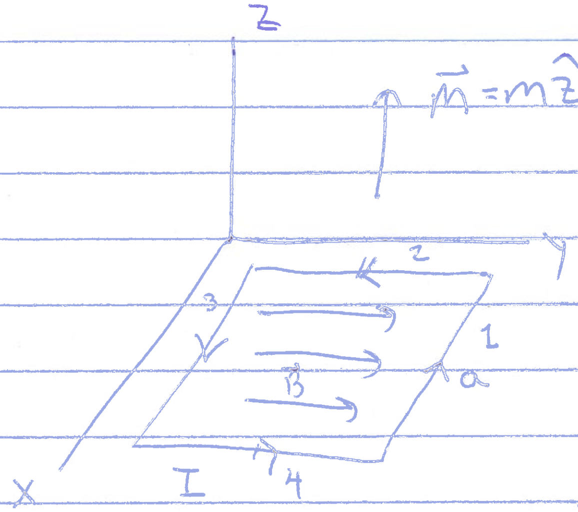
$\vec{F} = I\int d\vec{l} \times \vec{B}$

$\Rightarrow \vec{N} = IabB\sin\theta\,\hat{x} = mB\sin\theta\,\hat{x}$

$m = Iab$, $a \times b = ?$

$\Rightarrow \vec{N} = \vec{m} \times \vec{B} \rightarrow$ uniform field

6.2:7. $\rightarrow$ general loop



$\vec{B} = B\hat{y}$

$$\vec{F} = I\int d\vec{l}' \times \vec{B}$$

$$\vec{F}_{2,4} = 0 : d\vec{l}' \parallel \vec{B}$$

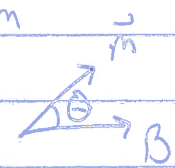
$$\vec{F}_1 = IaB(-\hat{z})$$

$$\vec{F}_2 = IaB(+\hat{z})$$

$$\vec{N}_1 = Ia\left(\frac{a}{2}\right)B(-\hat{x})$$

$$\vec{N}_2 = Ia\left(\frac{a}{2}\right)B(-\hat{x})$$

$$\vec{N} = \underbrace{Ia^2}_{m} B(-\hat{x})$$



$$\Rightarrow \boxed{\vec{N} = \vec{m} \times \vec{B}}$$

- Keep from rotating

$B_1 > B_3 \Rightarrow F_1 > F_2$, $F_{total} \to (-\hat{z}) \neq 0$

$$\Rightarrow \vec{F} = \nabla(\vec{m} \cdot \vec{B})$$

- paramagnetism seems like it would be universal since every electron is a dipole
- from QM, electrons are paired together, cancelling the effect.
- typically atoms with odd # of electrons are paramagnetic → one unpaired electron → silver atom in SG

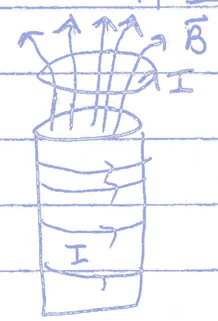
5s electron
orbit = 0
→ spin only

- Uniform field : $\vec{F} = I \oint (d\vec{l} \times \vec{B}) = I\left(\oint d\vec{l}\right) \times \vec{B}$

↖ $\vec{B}$ constant

$= 0$

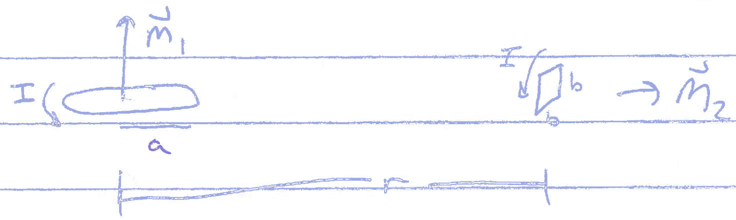
- non-uniform field $\vec{F} = 2\pi I R B \cos\theta$ → downward



- infinitesimal loop → $\vec{F} = \nabla(\vec{m} \cdot \vec{B})$

→ identical to force on a elec. dipole $\vec{F} = \nabla(\vec{p} \cdot \vec{E})$

$6.1 \rightarrow 270$



- torque exerted on the square loop due to circular loop

$r \gg a, b$

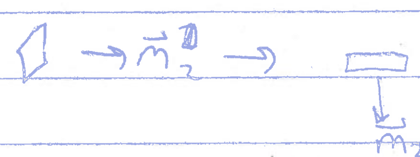
- $\vec{N} = \vec{m}_2 \times \vec{B}_1$, $\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3}\left[3(\vec{m}_1 \cdot \hat{r})\hat{r} - \vec{m}_1\right]$

$\hat{r} = \hat{y}$; $\vec{m}_1 = m_1 \hat{z}$, $\vec{m}_2 = m_2 \hat{y}$ $\Rightarrow$ $\vec{m}_1 \cdot \hat{r} = 0$

$\vec{B}_1 = -\frac{\mu_0}{4\pi} \frac{m_1}{r^3} \hat{z}$; $\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \cancel{I}m_2 & 0 \\ 0 & 0 & -\frac{\mu_0 m_1}{4\pi r^3} \end{vmatrix} = \hat{x}\left(-\frac{\mu_0 m}{4\pi r^3} I b^2\right)$

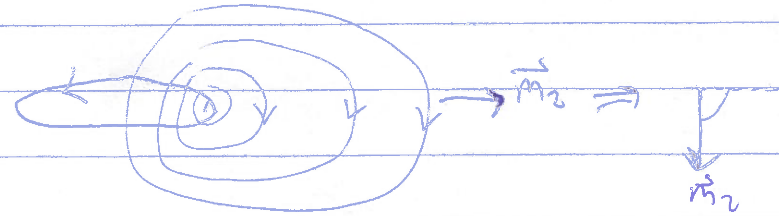
$\vec{N} = -\frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^3} (\hat{y} \times \hat{z}) = -\frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^3} \hat{x}$

$m_1 = \pi a^2 I$, $m_2 = b^2 I$



$\Rightarrow$ $\vec{N} = -\frac{\mu_0}{4} \frac{(abI)^2}{r^3} \hat{x}$

end Mon. Oct. 6.



- make sure I graded correctly!

- HW 6.5 (a,b)
  6.7
  6.12
  6.16
- Due Wed. Oct. 22
- quiz in class
  Fri. 17th

- Before curve
  66.5 average
  71 → median

90-100	4
80-89	4
65-79	8
50-64	6
< 50	7

$$\int (\nabla \times \vec{v}) \cdot d\vec{a} = \oint \vec{v} \cdot d\vec{l}$$

- Divergence theorem

$$\int \nabla \cdot \vec{v}\, d\tau = \oint \vec{v} \cdot d\vec{a}$$

- entire theory of E & M is based upon these equations — not something to skip

- Quizzes #3 average 57.9 %
  median 60 %

- 6.5 (a)+(b) ✓ old book!
- 6.7 ↖ ✓ ↖ $\vec{J}=J_0\hat{z}$
- 6.12 $\vec{K}=ks\hat{z}$ ✓
- 6.16 coax cable

66.5 → average
71 median
29 people

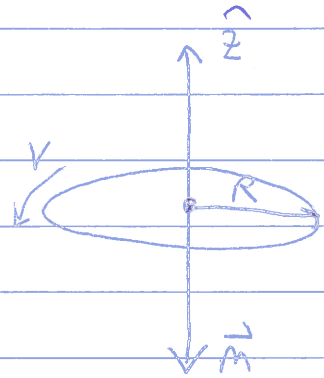
98 ↓ 25

A	4	90-100
B	4	80-89
C	7	70-79
D	5	60-69
F	9	0-59

Curve? +2

-quiz Friday

- Effect of B-field on atomic orbitals



$$T=\frac{2\pi R}{v}, \quad I=\frac{e}{T}=\frac{ev}{2\pi R}$$

$$m=I\pi R^2$$

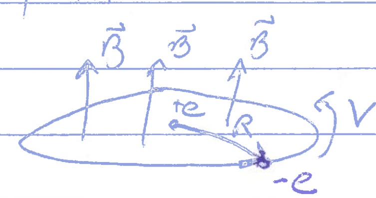
$$\vec{m}=-\frac{1}{2}evR\hat{z}$$

- because it is easier to tilt orbit as opposed to spin, orbital contribution to paramagnetism is small
- however, electron speeds up or slows down

$$(1) \quad \frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2}=m_e\underbrace{\left(\frac{v^2}{R}\right)}_{\text{centripetal accel.}}$$

→ now turn on B-field

$$F_{mag}=-e(\vec{v}\times\vec{B})$$



$$(2) \quad \frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2}+ev'B=\frac{m_e v'^2}{R}, \quad v' \rightarrow \text{in presence of } \vec{B}\text{-field}$$

$$(2)-(1) = ev'B=\frac{m_e}{R}(v'^2-v^2)=\frac{m_e}{R}(v'+v)(v'-v)$$

$\Delta v = v'-v \rightarrow$ small $\rightarrow$ $\boxed{\Delta v \approx \frac{eRB}{2m_e}}$

$$-\frac{eRB}{m_e}\left(\frac{v'}{v'+v}\right)=\Delta v$$

$v'=1.0000 \quad v=1.0001$

- $\Delta\vec{m} = -\frac{1}{2}e(\Delta v)R\,\hat{z} = \frac{-e^2R^2}{4m_e}\vec{B}$
- change in $\vec{m}$ opposite to direction of $\vec{B}$
- this is the mechanism of diamagnetism
  - small "extra" dipole moment antiparallel to field
- much weaker than paramagnetism, observed in atoms with even # of electrons
- classical interpretation (fundamentally flawed)

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* Magnetization: (magnetic polarization)

(1) Paramagnetism: dipoles associated w/ spins of unpaired electrons experience torque

(2) Diamagnetism: electron orbital speed changes to oppose applied field.

$\vec{M} \equiv$ magnetic dipole moment per volume

- both (1) + (2) are very weak
  - paramagnets attracted to field
  - diamagnets repelled by field
  - ferromagnets 10^4–10^5 as strong

-quiz

6.2.1

-colloquium Monday Oct. 20th room 321 @ 4:00PM astrophysics

...gnetized object

...a piece of material

...$\hat{\mathscr{r}}$ $\qquad \vec{M}$ = dipole moment per volume

...nt $d\tau'$

has dipole moment $\vec{M}\,d\tau'$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{M}(\vec{r}\,') \times \hat{\mathscr{r}}\, d\tau'}{\mathscr{r}^2}$$

$$\Rightarrow \nabla' \frac{1}{\mathscr{r}} = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\Rightarrow \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \left[\vec{M}(\vec{r}\,') \times \left(\nabla' \frac{1}{\mathscr{r}}\right)\right] d\tau'$$

-p.r. #7

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\left[\int \frac{1}{\mathscr{r}}\left[\nabla' \times \vec{M}(\vec{r}\,')\right] d\tau' - \int \nabla' \times \left[\frac{\vec{M}(\vec{r}\,')}{\mathscr{r}}\right] d\tau'\right]$$

$$\int (\nabla \times \vec{v})\, d\tau = -\oint \vec{v} \times d\vec{a}$$

$$\Rightarrow \frac{\mu_0}{4\pi} \int \nabla' \times \left[\frac{\vec{M}(\vec{r}\,')}{\eta}\right] d\tau' = \frac{\mu_0}{4\pi} \oint \frac{1}{\eta} \left[\vec{M}(\vec{r}\,') \times d\vec{a}\,'\right]$$

$$\Rightarrow \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{1}{\eta} \left[\nabla' \times \vec{M}(\vec{r}\,')\right] d\tau' + \frac{\mu_0}{4\pi} \oint \frac{1}{\eta} \left[\vec{M}(\vec{r}\,') \times d\vec{a}\,'\right]$$

$$\vec{J}_b = \nabla \times \vec{M} \quad , \vec{K}_b = \vec{M} \times \hat{n}$$

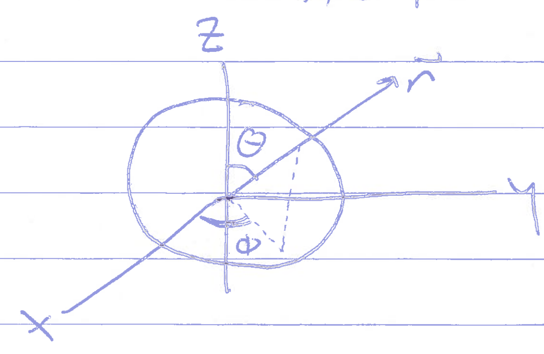
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{Vol} \frac{\vec{J}_b(\vec{r}\,')\, d\tau'}{\eta} + \frac{\mu_0}{4\pi} \oint_S \frac{\vec{K}_b(\vec{r}\,')\, da'}{\eta} \qquad 6.22$$

- go from adding infinitesimal dipoles to surface & volume current?

- uniformly magnetized sphere $\vec{M} = M\hat{z}$

$$\vec{J}_b = \nabla \times \vec{M} = 0 \rightarrow \text{uniform}$$

$$\vec{K}_b = \vec{M} \times \hat{n} = M \sin\theta\, \hat{\phi}$$



$\vec{K} = \sigma \vec{v} = \sigma \omega R \sin\theta\, \hat{\phi}$, rotating spherical shell

$\sigma R \vec{\omega} \rightarrow \vec{M}$ identical

$\vec{B} = \frac{2}{3} \mu_0 \sigma R \vec{\omega} \rightarrow \vec{B} = \frac{2}{3} \mu_0 \vec{M}$ inside

$\vec{m} = \frac{4}{3} \pi R^3 \vec{M}$ outside

6.8:

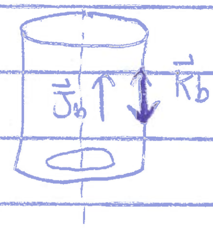
$\vec{M} = ks^2\hat{\phi}$

$$\nabla\times\vec{M} = J_b = \frac{1}{s}\left[\frac{\partial s(ks^2)}{\partial s}\right]\hat{z} = \frac{1}{s}\cdot 3ks^2 = \boxed{3ks\hat{z}}$$

$$\vec{K}_b = \vec{M}\times\hat{n} = \begin{vmatrix}\hat{s} & \hat{\phi} & \hat{z}\\ 0 & ks^2 & 0\\ 1 & 0 & 0\end{vmatrix} = -ks^2\hat{z}$$

$ks^2 \to kR^2$

$$\boxed{\vec{K}_b = -kR^2\hat{z}}$$

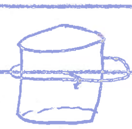


$$s\le R:\ \oint \vec{B}\cdot d\vec{l} = B\cdot 2\pi s = \mu_0 I_{enc} = \mu_0\int \vec{J}_b\cdot d\vec{a}$$

$$= \mu_0\cdot 2\pi\, 3k\int_0^s s'(ds'\,s') = \mu_0 2\pi k s^3 = B\cdot 2\pi s$$

$$\to \vec{B} = \overset{=\mu_0\vec{M}}{\mu_0 ks^2\hat{\phi}} \ :\ s\le R$$

$s\ge R$:



$$B\cdot 2\pi s = \mu_0 I_{enc} = \mu_0\left[\int_0^{2\pi}\int_0^R 3ks(s\,ds)\,d\phi - \int_0^{2\pi} +kR^2(R\,d\phi)\right]$$

$$= 2\pi\mu_0\left[kR^3 - kR^3\right] = 0 \to \vec{B} = 0 \text{ out}$$

- Magnetization is responsible for setting up bound currents $\vec{J}_b = \nabla \times \vec{M}$ + surface currents $\vec{K}_b = \vec{M} \times \hat{n}$

→ we did not mention where these came from

$$\vec{J} = \vec{J}_b + \vec{J}_f$$

← current flowing say from battery etc.

- bound current only present due to magnetization

$$\nabla \times \vec{B} = \vec{J} = \vec{J}_f + \vec{J}_b = \vec{J}_f + (\nabla \times \vec{M})$$

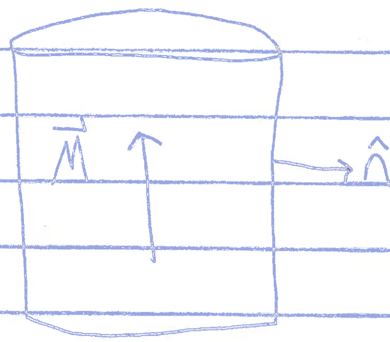
$$\Rightarrow \nabla \times \underbrace{\left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right)}_{\vec{H}} = \vec{J}_f \quad , \quad \boxed{\vec{B} = \mu_0(\vec{H} + \vec{M})}$$

end Wed.

$$\nabla \times \vec{H} = \vec{J}_f \rightarrow \oint \vec{H} \cdot d\vec{l} = I_{f\,enc}$$

- we control $\vec{J}_f$, so more convenient form

- Magnetization establishes volume current & surface current (currents are bound)



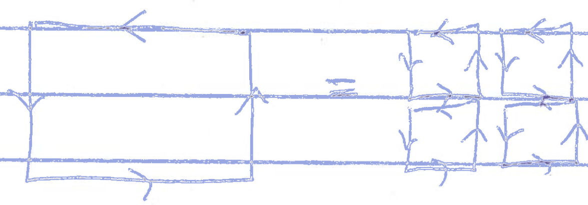
$\vec{J}_b = \nabla \times \vec{M} = 0 \rightarrow$ uniform $\vec{M}$

$$\vec{M} \times \hat{n} = \begin{vmatrix} \hat{s} & \hat{\phi} & \hat{z} \\ 0 & 0 & M \\ 1 & 0 & 0 \end{vmatrix} = \boxed{M\hat{\phi} = K_b}$$

$\rightarrow$ solenoid

$\vec{B} = \mu_0 I n \hat{z}$, $n = \frac{\text{turns}}{\ell}$

$K = \frac{dI}{dl_\perp}$



$\rightarrow \vec{B} = \mu_0 K_b \hat{\phi} = \mu_0 M \hat{\phi}$

non-uniform $\rightarrow$ currents don't cancel $\rightarrow$ volume current density

Begin Friday Oct. 17th
- quiz #4 after 20 minutes

$\vec{J} = \vec{J}_b + \vec{J}_f$

$\frac{1}{\mu_0}(\nabla \times \vec{B}) = \vec{J} = \vec{J}_b + \vec{J}_f = (\nabla \times \vec{M}) + \vec{J}_f$

$\Rightarrow \nabla \times \left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right) = \vec{J}_f \quad , \quad \vec{B} = \mu_0 (\vec{H} + \vec{M})$

$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \Leftrightarrow \boxed{\nabla \times \vec{H} = \vec{J}_f}$

- write ~~H~~ in terms of free current alone
Ampère's law

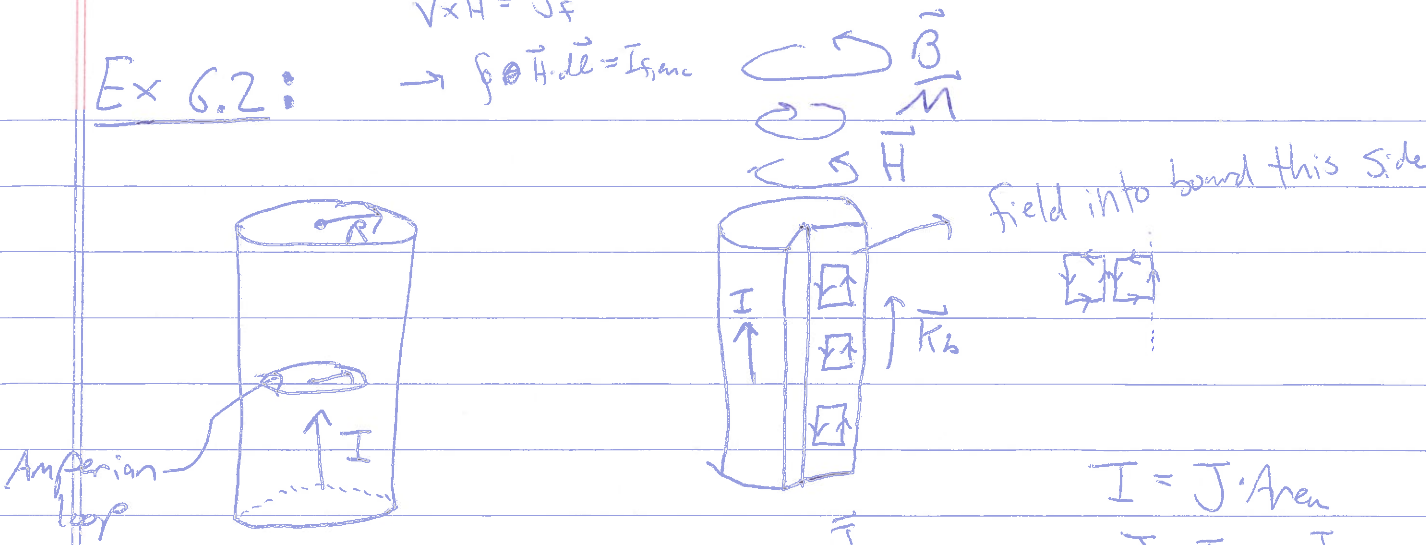
$\oint \vec{H} \cdot d\vec{l} = I_{f,enc}$

- begin monday Oct. 20th

$\nabla \times \vec{H} = \vec{J}_f$

Ex 6.2:

$\rightarrow \oint \vec{H} \cdot d\vec{l} = I_{f,enc}$



$I = J \cdot Area$

$J = \frac{I}{Area} = \frac{I}{\pi R^2}$

$J \cdot \pi s^2 = I_{enc}$

$= \frac{I}{\pi R^2} \cdot \pi s^2$

$s \le R$: $\quad H(2\pi s) = I_{f\,enc} = \frac{I \pi s^2}{\pi R^2}$

$$\vec{H} = \frac{I}{2\pi R^2} s \hat{\phi}$$

$I_{enc} = I$

$$s \ge R: \vec{H} = \frac{I}{2\pi s} \hat{\phi}$$

never magnetization w/ no material

$$s \ge R: \quad \vec{M} = 0 \rightarrow \vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I}{2\pi s} \hat{\phi}$$

$\vec{B} = \mu_0 (\vec{H} + \vec{M})$

$\vec{B}$ for $s \le R \rightarrow$ comes later

- we don't know $\vec{M}$ yet. small for typical wire material Cu

*Note: $I_{enc} = \int \vec{J} \cdot d\vec{a}$, $s \ge R \rightarrow I_{enc} = I = J \cdot \pi R^2 \rightarrow J = \frac{I}{\pi R^2}$

$s \le R$: $I_{enc} = J \int da = J \cdot \pi s^2 = \frac{I \pi s^2}{\pi R^2}$ ✓

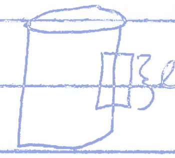
Linear Media:

$\vec{M} = \chi_m \vec{H}$, $\chi_m \equiv$ magnetic susceptibility

Au : -3.4×10^{-5} , minus diamagnetic

Liquid O_2 : 3.9×10^{-3}, + paramagnetic

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \underbrace{\mu_0 (1 + \chi_m)}_{\mu} \vec{H}$$



$H \cdot l = I n l$

$\Rightarrow \vec{H} = n I \hat{z}$

- Solenoid : $\vec{H} = n I \hat{z}$

$$\vec{B} = \underbrace{\mu_0 (1 + \chi_m)}_{\mu} \underbrace{n I \hat{z}}_{\vec{H}}$$

→ vacuum : $\vec{B} = \mu_0 n I \hat{z}$

enhanced if paramag.

decreased if diamagnetic

$\nabla\cdot\vec H = \frac{1}{\mu_0}\{\nabla\cdot\vec B - \nabla\cdot\vec M\}$ → only situations where $\nabla\cdot\vec M$ also 0 does Ampère's law hold.

$\Rightarrow \nabla\cdot\vec H = -\nabla\cdot\vec M$

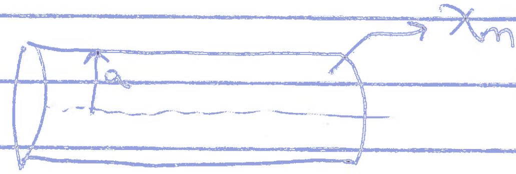
- Boundary conditions:

$$H^\perp_{above} - H^\perp_{below} = -\left(M^\perp_{above} - M^\perp_{below}\right)$$

$$\nabla\times\vec H = \vec J_f \Rightarrow$$

$$\vec H^\parallel_{above} - \vec H^\parallel_{below} = \vec K_f \times \hat n$$

* 6.17:



$$\oint \vec H\cdot d\vec l = H(2\pi s) = I_{f,enc} = \begin{cases} I(s^2/a^2); & s<a \\ I; & s>a\end{cases}$$

$\chi_m<1$ → reduced field

$$\vec H = \begin{cases}\frac{Is}{2\pi a^2}\hat\phi & s<a \\ \frac{I}{2\pi s}\hat\phi & s>a\end{cases};\quad \vec B = \underset{\text{linear material}}{\mu} H = \begin{cases}\frac{\mu_0(1+\chi_m)Is}{2\pi a^2}\hat\phi & s<a\\ \frac{\mu_0 I}{2\pi s}\hat\phi & s>a\end{cases}$$

$$\vec J_b = \nabla\times\vec M = \nabla\times(\chi_m \vec H) = \chi_m \vec J_f$$

linear media, $\nabla\times\vec H = \vec J_f$ → Ampère's law

$$\vec J_f = \frac{I}{\pi a^2}\hat z;\quad \vec J_b = \frac{\chi_m I}{\pi a^2}\hat z,\quad \vec K_b = \vec M\times\hat n = \chi_m \vec H\times\hat n = \frac{\chi_m I}{2\pi a}(-\hat z)$$

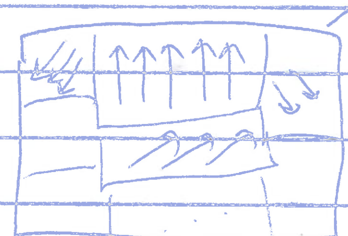
$\hat\phi\times\hat s = (-\hat z)$

Eli → 2
Zach → 1

→ Q

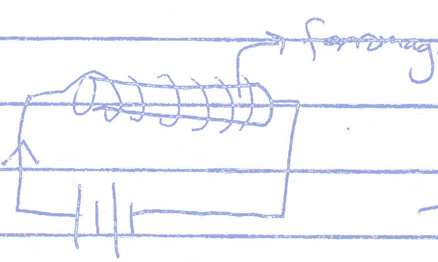
Ferromagnetism: natural elements Co, Ni, Fe

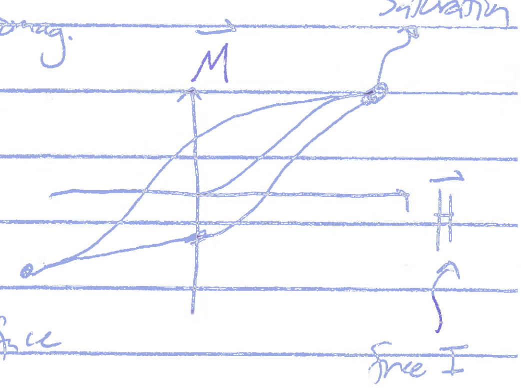
- For linear media, alignment of dipoles is sustained by the external field.
- For ferromagnets (very non-linear) require no external field
- arrises from spins of unpaired electrons
- Q.M. by nature
- dipoles like to be aligned to their neighbors
- alignment occurs in domains, but they are randomly oriented



- in external field $\vec{B}$ → $\vec{N} = \vec{m} \times \vec{B}$ → aligns dipoles

- if field is strong enough → 1 domain remains "saturated"





- Magnetic Shielding: