* Will call out students in class

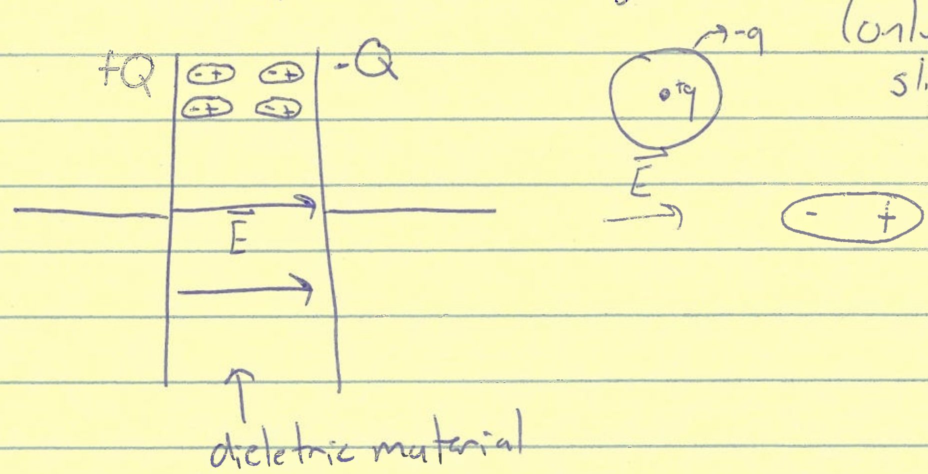
- talk about structure of class, hand out sylabi:
- may spend short times on Ch. 4 + 6.
- very important are Ch. 5, Ch. 7 + some of Ch. 9.

- Electric fields in Matter:

- most materials are dielectric (elec. ins.)
  - glass, wood, plastics
  -

- in contrast to conductors - studied in Semester I
- semiconductors are somewhere between - not studied here (maybe solid state)

- dielectrics can be polarized, charges are fixed (only ~~move~~ slightly)



Q - what would happen with conductor?

- even though the atom is neutral overall, it consits of equal + & - charges
- if the field is large enough, ionization occurs.

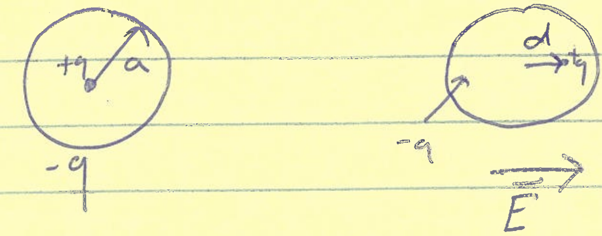
$$\vec{p} = \alpha \vec{E} \qquad \text{(linear, low fields)}$$

dipole moment

$$\alpha_H = 0.667 \; ; \; \alpha_{He} = 0.205$$

$\alpha \equiv$ atomic polarizability

Ex. 4.5



Q · what is the dipole moment? (in terms of q, d)

$$p = qd$$

· internal field pushes nucleus to left

$E_e \equiv$ field of electron cloud

$E = E_e \rightarrow E_e = \frac{1}{4\pi\epsilon_0}\frac{qd}{a^3}$ → prob. 2.12

$\rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{qd}{a^3} = \frac{1}{4\pi a^3 \epsilon_0} p$

$\Rightarrow p = (4\pi\epsilon_0 a^3) E$

$\Rightarrow \alpha = 4\pi\epsilon_0 a^3$, what is $\frac{4}{3}\pi a^3$? $= V$

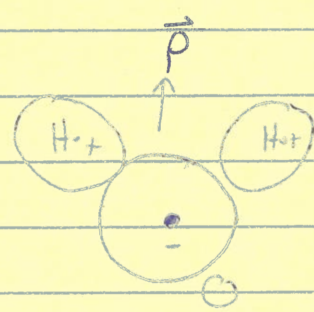
$\Rightarrow \alpha = 3\alpha V$ (roughly correct)

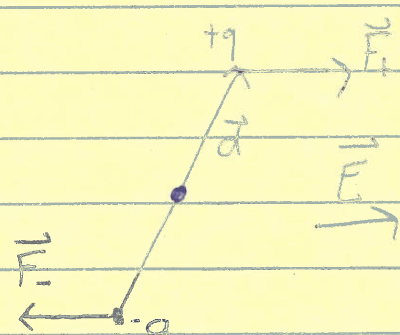
- permanent dipole moment

$\vec F_+ = q\vec E$

Q: what is a good polar molecule?

$\vec F_- = -q\vec E$





$\vec N_+ = \vec r_+ \times \vec F_+$
$= (\vec d/2 \times q\vec E)$

$\vec N_- = \vec r_- \times \vec F_-$
$= (-\vec d/2 \times (-q\vec E)) \Rightarrow \vec N = q\vec d \times \vec E = \vec p \times \vec E$

$\vec N = \vec N_+ + \vec N_-$

quiz: 4.4.1

- Microwave oven example, H_2O

~~end Monday: 08.25.14~~

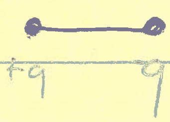
Wednesday: 08.27.14 H.W. $4.\cancel{8}$ 4.4, 4.10, 4.15, $4.2\cancel{0}$1,

- Force on dipole: more 4 pages op.

$\boxed{\text{problem: 4.7} \quad U = -\vec{p}\cdot\vec{E} \text{, energy } U \text{ ; point charge } W = QV(\vec{r}) \quad \text{- physical dipole } -q \text{ at } \vec{r} \text{ \& } +q \text{ at } \vec{r}+\vec{d}}$

X

- Force on dipole: $\vec{F} = \vec{F}_+ + \vec{F}_- = q(\vec{E}_+ - \vec{E}_-)$
$= q(\Delta E)$



eq. 1.35, if dipole is short $\Delta E_x \equiv (\nabla E_x)\cdot \vec{d}$

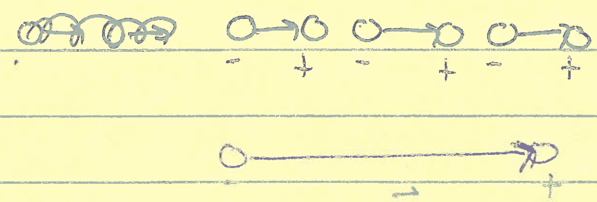
$\Rightarrow \Delta \vec{E} = (\vec{d}\cdot\vec{\nabla})\vec{E}$

$\Rightarrow \vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E}$

$* \ \vec{F} = \nabla(\vec{p}\cdot\vec{E})$

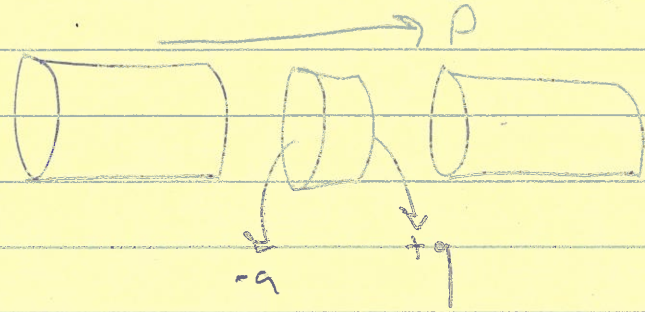
1.35 $dT = \nabla T \cdot d\vec{l}$

Polarization: $P \equiv$ dipole moment per volume



$$\vec{P} = \frac{\langle \vec{p} \rangle}{V}$$

- not thinking of cause, but what field a polarized object produces

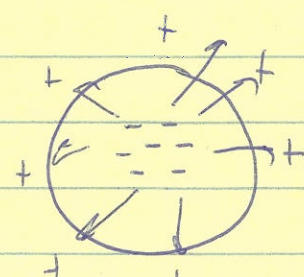


$$\boxed{\begin{aligned} p &= P(A \cdot d) \\ qd &= P \cdot (A \cdot d) \\ q &= P \cdot A \end{aligned}}$$

$\Rightarrow$, $\sigma_b = \frac{q}{A} = P$

$\vec{P} \cdot \hat{n}$, $\sigma_b = \frac{q}{A_{end}} = P\cos\theta = \vec{P} \cdot \hat{n}$

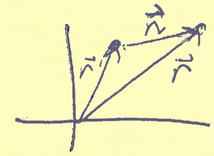
- for non uniform polarization, charge accumulates in the material



$\int \rho_b \, d\tau \rightarrow$ net bound charge $\rightarrow$ same as pushed out through surface

$$\int_{Vol} \rho_b \, d\tau = -\oint_S \vec{P} \cdot d\vec{a} = -\int_{Vol} (\vec{\nabla} \cdot \vec{P}) \, d\tau \Rightarrow \rho_b = -\nabla \cdot \vec{P}$$

- the net bound charge is equal & opposite to the amount pushed through the surface



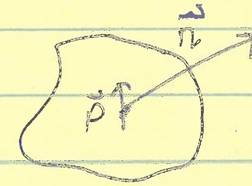
$\vec{r}' + \vec{\mathscr{r}} = \vec{r}$

The field of a Polarized object

Section 4.2.1

- potential of a dipole (single)

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{\mathscr{r}}}{\mathscr{r}^2}$$



$$\vec{p} = \vec{P}\, d\tau'$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\vec{P}(\vec{r}') \cdot \hat{\mathscr{r}}\, d\tau'}{\mathscr{r}^2}$$

*trick: $\nabla'\left(\frac{1}{\mathscr{r}}\right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$ → respect to source

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0} \int_V \vec{P} \cdot \nabla'\left(\frac{1}{\mathscr{r}}\right) d\tau'$$

- int. by parts (product rule #5)

$$V = \frac{1}{4\pi\epsilon_0}\left[\int_{vol} \nabla' \cdot \left(\frac{\vec{P}}{\mathscr{r}}\right) d\tau' - \int_{vol} \frac{1}{\mathscr{r}} (\nabla' \cdot \vec{P})\, d\tau'\right]$$

- div. theorem

$$\frac{1}{4\pi\epsilon_0} \oint \frac{1}{\mathscr{r}} \vec{P} \cdot d\vec{a}' - \frac{1}{4\pi\epsilon_0} \int_{vol} \frac{1}{\mathscr{r}} (\nabla' \cdot \vec{P})\, d\tau'$$

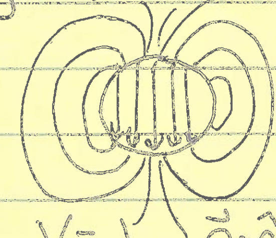
$\sigma_b = \vec{P} \cdot \hat{n}$ $\qquad$ $\rho_b = -\nabla \cdot \vec{P}$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\oint_S \frac{\sigma_b\, da'}{r} + \frac{1}{4\pi\epsilon_0}\int \frac{\rho_b\, d\tau'}{r}$$

*potential of a polarized object

- we can determine the field by looking at only the bound charges & the fields the produce.
- simplifies calculation (Gauss law etc.)
- already said before that charges are actually real.

-Go through ex. 4.2



$$V = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

- Same as perfect dipole at the origin

- Electric Displacement

- if we have not only bound charge caused by the field, but also free charges (not bound to any single atom)

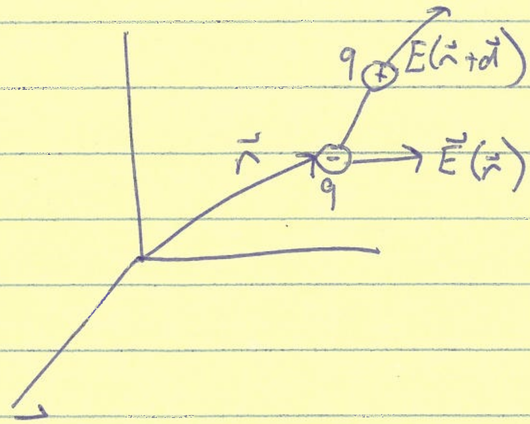
- total charge density:

$$\rho = \rho_b + \rho_f$$

- Gauss' law: $\epsilon_0 \vec{\nabla}\cdot\vec{E} = \rho = \rho_b + \rho_f = -\nabla\cdot\vec{P} + \rho_f$

- $\vec{E}$ is the total field

Force on a dipole



$$\boxed{dT = \nabla T \cdot d\vec{l}}$$

$$\vec{F} = q\vec{E}$$

$$\lim_{\substack{q \to \infty \\ d \to 0}} (qd) = p \quad , \quad \vec{F} = q\left[\vec{E}(\vec{r}+\vec{d}) - \vec{E}(\vec{r})\right]$$

$$\vec{F} = q\Delta\vec{E}$$

$$F_x = q\left[E_x(x+d_x, y+d_y, z+d_z) - E_x(x,y,z)\right]$$

$$= q\left[E_x(x,y,z) + d_x \frac{\partial E_x}{\partial x} + d_y \frac{\partial E_x}{\partial y} + d_z \frac{\partial E_x}{\partial z} + \ldots - E_x(x,y,z)\right]$$

- limit d is small on the order of distances over which E varies appreciably

$$F_x = \vec{p} \cdot \nabla E_x$$

$$\vec{F} = \vec{p} \cdot \nabla \vec{E}$$

$$\Rightarrow \nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_f$$

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_f \quad \text{or} \quad \oint \vec{D} \cdot d\vec{a} = Q_{f_{enc}}$$

*Note $\vec{D}$ is not exclusively determined by free charge

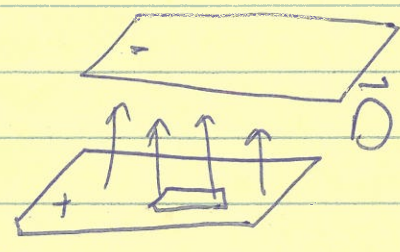
$$\nabla \times \vec{D} = \epsilon_0 \underbrace{\cancel{\nabla \times \vec{E}}}_{0} + \nabla \times \vec{P}$$

electrostatics

- bar electret, electric analog of bar magnet

ex: displacement in capacitor

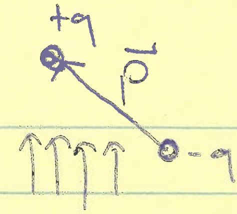
-later



$$\oint \vec{D} \cdot d\vec{A} = Q_{free}$$

$$|\vec{D}| = \frac{Q_f}{A}$$

- beginning of Wed. Class



$\vec{E} = E_0\hat{z}$

$\vec{N} = \vec{p}\times\vec{E}$

$= Ep\sin\theta$

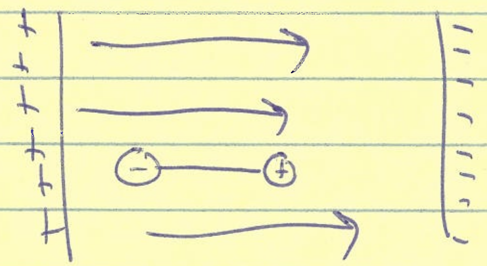
- prob. 4.7

$$U = \int_0^{\theta} N\, d\theta' = \int_0^{\theta} Ep\sin\theta'\, d\theta'$$

$$U = -Ep\cos\theta \Big|_0^{\theta} = -Ep\cos\theta + Ep$$

$$U = 0 \ @ \ \theta = 90^\circ$$

$$U = -Ep\cos\theta = -\vec{p}\cdot\vec{E}$$



low energy

high energy
(unstable

Page II

Friday: 08.29.14

* $\vec{P}\cdot d\vec{a} = (\vec{P}\cdot\hat{n})da$

$\rho_b = -\nabla\cdot\vec{P}$

$\sigma_b = \vec{P}\cdot\hat{n}$

Prob: 4.14, prove, $Q_{total,\,bound} = 0$

pg. 179 $\sigma_b \equiv \vec{P}\cdot\hat{n}$, $\rho_b \equiv -\nabla\cdot\vec{P}$

$$Q_{tot} = \oint_S \sigma_b da + \int_{vol} \rho_b d\tau$$

$$= \oint_S \vec{P}\cdot d\vec{a} - \int_{vol} \nabla\cdot\vec{P} d\tau$$

div. theorem: $\oint_S \vec{P}\cdot d\vec{a} = \int_{vol} \nabla\cdot\vec{P} d\tau \Rightarrow Q = 0$ ✓

- within the dielectric $\rho = \rho_b + \rho_f$

- Gauss's law: $\epsilon_0 \nabla\cdot\vec{E} = \rho = \rho_b + \rho_f = -\nabla\cdot\vec{P} + \rho_f$

 - $\vec{E}$ is the total field

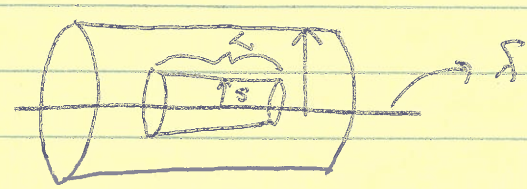
$\nabla\cdot(\epsilon_0\vec{E} + \vec{P}) = \rho_f \Rightarrow \vec{D} = \epsilon_0\vec{E} + \vec{P}$

- Gauss's law $\nabla\cdot\vec{D} = \rho_f$ or $\oint \vec{D}\cdot d\vec{a} = Q_{f\,enc}$

$Q_{f\,enc} \equiv$ total free charge enclosed in volume

- we usually know ρ_f in a problem, but not ρ_b

Ex 4.4



$$\oint \vec{D}\cdot d\vec{a} = D(2\pi s L) = \lambda L$$

$\Rightarrow \vec{D} = \dfrac{\lambda}{2\pi s}$, outside the insulation $\vec{P} = 0 \Rightarrow E = \dfrac{1}{\epsilon_0} D = \dfrac{\lambda}{2\pi\epsilon_0 s}\hat{s}$

- inside we don't know E, since we don't know $\vec{P}$

- exams out of class?

Page I

- why are we interested in this topic?
  - in particular why dipoles?

- dipole in total charge is 0. An electricall neutral piece of material seems like it would not be affected by EM fields, but we know this is not true.

- Example would be curved glass, or lensing. Your eyeglasses are not charged → there are no monopoles, yet they bend light (EM radiation)
- this is because the material is made of charged individual particles, electrons & neutrons. It is really the electrons which are responsible for refraction.

- This is why dipoles are always talked about in E&M.

- Discrete dipole approximation → computing scattering of radiation by particles of arbitrary shape.
- proposed for larger particles in 1973 to study scattering effects of interstellar dust grains.
- finite array of polarizable points & the dipoles interact with one another.

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{vol} \frac{\vec{P}(\vec{r}')\cdot\hat{r}\,d\tau'}{r^2}$$

$\vec{p} = \vec{P}\,d\tau'$

* only time I will write small $\vec{p}$

- there is no "Coulomb's law" for $\vec{D}$

$$\vec{D}(\vec{r}) \neq \frac{1}{4\pi}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\,\rho_f(\vec{r}\,')\,d\tau'$$

- one must know both the divergence & curl of a field in order to solve it.

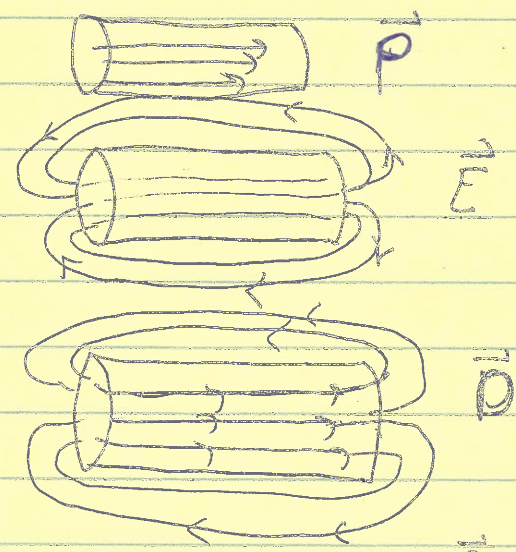
$$\nabla\times\vec{D} = \epsilon_0(\nabla\times\vec{E}) + (\nabla\times\vec{P})$$

- for symmetric case (where Gauss's law works)

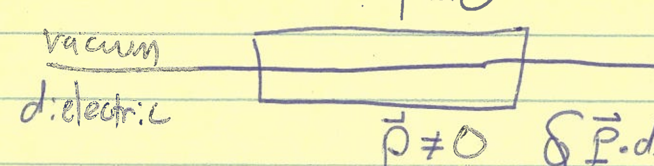
$\vec{\nabla}\times\vec{P} = 0$, in general $\nabla\times\vec{P} \neq 0$

- electret:

$\vec{D} = \epsilon_0\vec{E} + \vec{P}$



$\vec{P} = 0$



$\vec{P} \neq 0$ $\oint \vec{P}\cdot d\vec{l} \neq 0$

$\Leftrightarrow \nabla\times\vec{P} \neq 0$ everywhere

- Linear Dielectrics:

- what is the cause of polarization?

- for linear dielectrics $\vec{P} = \epsilon_0 \chi_e \vec{E}$

$\chi_e \equiv$ electric susceptibility (property of medium)
- depends upon microscopic structure.
- how easily a substance is polarized

- note that $\vec{E}$ is the total field, that means if the dielectric is placed in $\vec{E}_0$, you cannot compute $\vec{P}$.

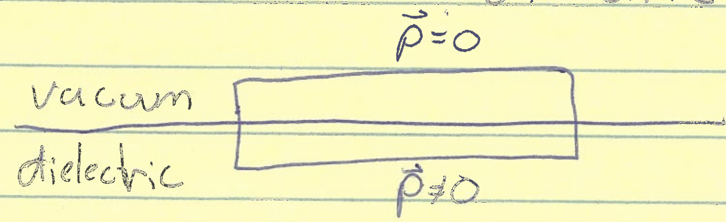
- we can look at the displacement

$$D = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 E + \epsilon_0 \chi_e \vec{E} = \epsilon_0 (1 + \chi_e) \vec{E}$$

$$D = \epsilon \vec{E}, \quad \epsilon \equiv \epsilon_0 (1 + \chi_e)$$

$\epsilon \equiv$ permittivity ; vaccum $\chi_e = 0$
$\epsilon = \epsilon_0$

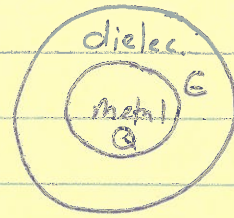
$\epsilon_r \equiv 1 + \chi_e = \dfrac{\epsilon}{\epsilon_0}$, relative permittivity or dielectric constant



$\oint \vec{P} \cdot d\ell \neq 0$

$\Leftrightarrow \nabla \times P \neq 0$ everywhere

Ex: 4.5



- inner radius a
- outer " b

- dielectric is linear

- we must have $\vec{E}$ to calculate $\vec{P}$, but we don't know it
- we do have knowledge of free charge Q_f

- due to symmetry we can use

$$\oint \vec{D}\cdot d\vec{a} = Q_{f\,enc} \Rightarrow D\cdot 4\pi r^2 = Q_{f\,enc}$$

$$\Rightarrow \vec{D} = \frac{Q_f}{4\pi r^2}\hat{r}, \; r > a$$

Q: What is $\vec{E}, \vec{D}$ inside metal sphere?

$$\vec{D} = \epsilon \vec{E} \Rightarrow \vec{E} = \begin{cases} \dfrac{Q}{4\pi\epsilon r^2}\hat{r}\,; & a < r < b \\ \dfrac{Q}{4\pi\epsilon_0 r^2}\hat{r}\,; & r > b \end{cases}$$

- find the potential:

$$V = -\int_{\infty}^{0} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{b}\left(\frac{Q}{4\pi\epsilon_0 r^2}\right)dr - \int_{b}^{a}\left(\frac{Q}{4\pi\epsilon r^2}\right)dr - \int_{a}^{0}(0)$$

$$= \frac{Q}{4\pi}\left(\frac{1}{\epsilon_0 b} + \frac{1}{\epsilon a} - \frac{1}{\epsilon b}\right)$$

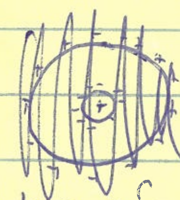
$$\vec{\nabla}\cdot\vec{P} = \frac{1}{r^2}\left(\frac{\partial}{\partial r} \cdots r^2 E_r\right) = 0$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E} = \frac{\epsilon_0 \chi_e Q}{4\pi\epsilon r^2}\hat{r}$$

$$\Rightarrow \rho_b = -\nabla\cdot\vec{P} = 0 \;;\; \sigma_b = \vec{P}\cdot\hat{n} = \begin{cases} \dfrac{\epsilon_0 \chi_e Q}{4\pi\epsilon b^2} & \text{outer} \\ \dfrac{-\epsilon_0 \chi_e Q}{4\pi\epsilon a^2} & \text{inner} \end{cases}$$

- layer between conductor and dielectric reduces the field by a factor

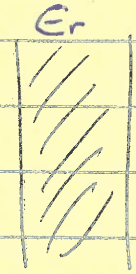
$$\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \text{ to } \frac{1}{4\pi\epsilon r^2}Q\,\hat{r}$$



- like imperfect conductor, partial cancelation of field.

Ex: capacitor with dielectric filling

ϵ_r



$$C = \frac{Q}{V}, \qquad \boxed{\begin{aligned} V &= \frac{Q}{A\epsilon_0}d \\ V &= E\cdot d \end{aligned}}$$

- field is reduced, so too is V by $\frac{1}{\epsilon_r}$

$\boxed{C = \epsilon_r C}$, which increases capacitance

- infinite plate

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}$$

- capacitor $\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$

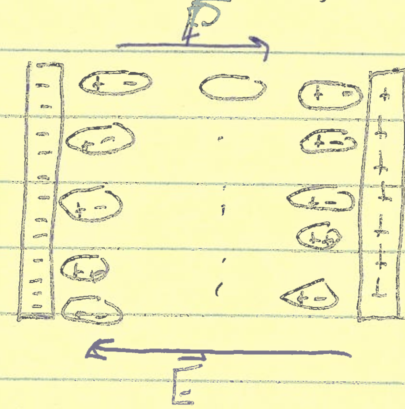
$$W = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}VQ$$

- Energy in dielectrics

  - work to charge a capacitor: $W = \frac{1}{2}CV^2$

    - with dielectric $C = \epsilon_r C_{vac}$

- one must add more charge to acheive the same potential because field is partially cancelled



- from Ch. 2 $W = \frac{\epsilon_0}{2}\int E^2 d\tau$

  - for dielectric-filled capacitor

$$W = \frac{\epsilon_0}{2}\int \epsilon_r E^2 d\tau = \frac{1}{2}\int \vec{D}\cdot\vec{E}\,d\tau$$

- proof: bring in ρ_f a little at a time

$$\rightarrow \Delta\rho_f \rightarrow \Delta W = \int (\Delta\rho_f)\,V\,d\tau \;;\; W = QV(\vec{r})$$

- $\nabla\cdot D = \rho_f$, $\Delta\rho_f = \nabla\cdot(\Delta\vec{D})$

$$\Delta W = \int [\nabla\cdot(\Delta\vec{D})\,V\,d\tau$$

$$\nabla \cdot [(\Delta \vec{D}) V] = [\nabla \cdot (\Delta \vec{D})] V + \Delta \vec{D} \cdot (\nabla V)$$

$$\Rightarrow \Delta W = \int \nabla \cdot [(\Delta \vec{D}) V] \, d\tau + \int (\Delta \vec{D}) \cdot E \, d\tau$$

$E = -\nabla V$

$\Rightarrow$ 1^{st} integral vanishes @ ∞ (surface)

$$\Rightarrow \Delta W = \int (\Delta \vec{D}) \cdot \vec{E} \, d\tau$$

$\vec{D} = \epsilon \vec{E}$ $\rightarrow$ linear dielectrics

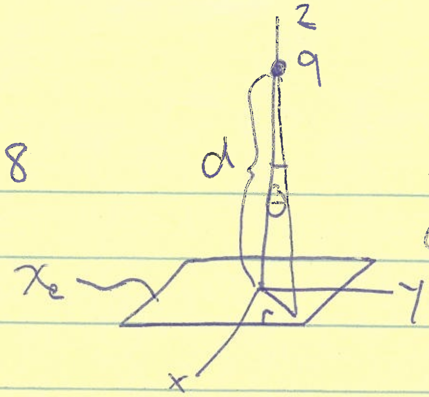
$$-\ \frac{1}{2} \Delta (\vec{D} \cdot \vec{E}) = \frac{1}{2} \Delta (\epsilon E^2) = \epsilon (\Delta \vec{E}) \cdot \vec{E}$$

$$= (\Delta D) \cdot \vec{E}$$

III.

Pg. 194

Ex: 4.8



$r = \sqrt{x^2+y^2}$

$\cos\theta = d / \sqrt{d^2+r^2}$

- surface bound charge is opposite to q → attractive

$\sigma_b = \vec{P}\cdot\hat{n} = P_z = \epsilon_0 \chi_e E_z$

$E_{z,q} = \frac{-1}{4\pi\epsilon_0} \frac{q}{(r^2+d^2)} \cos\theta = -\frac{1}{4\pi\epsilon_0} \frac{qd}{(r^2+d^2)^{3/2}}$

$E_{z,b} = \frac{-\sigma_b}{2\epsilon_0}$ → field due to bound charge

$\sigma_b = \epsilon_0 \chi_e \left[\frac{-1}{4\pi\epsilon_0} \frac{qd}{(r^2+d^2)} - \frac{\sigma_b}{2\epsilon_0} \right]$

$\rightarrow \sigma_b = \frac{-1}{2\pi} \left(\frac{\chi_e}{\chi_e+2} \right) \frac{qd}{(r^2+d^2)^{3/2}}$ → on dielectric

- conductor (surface) $\sigma = -\frac{qd}{2\pi} \cdot \frac{1}{(r^2+d^2)^{3/2}}$ - q produces this bound charge on conduc.

$\Rightarrow \boxed{q_b = -\left(\frac{\chi_e}{\chi_e+2}\right) q}$, $\chi_e = \epsilon_r - 1$

- H_2O vapor → $\chi_e = 0.006$

$\left(\frac{\chi_e}{\chi_e+2}\right) \simeq 3 \times 10^{-3}$

Q: - conductor $\chi_e \to \infty$; $\left(\frac{\chi_e}{\chi_e+2}\right) \to$?

- corollary: uniq. theorem — pot. is unique if (a) charge density (b) value of V is known on all boundaries

- Method of images: ~~2nd uniqueness theorem — total charge on each conductor is given~~

- for a single point charge q_b @ $(0,0,-d)$

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{(x^2+y^2+(z-d)^2)^{1/2}} + \frac{q_b}{(x^2+y^2+(z+d)^2)^{1/2}}\right] ; z>0$$

→ next page

- a charge of $(q+q_b)$ @ $(0,0,d)$ gives

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q+q_b}{(x^2+y^2+(z-d)^2)^{1/2}}\right] ; z<0$$

$\nabla^2 V = \rho/\epsilon_0$

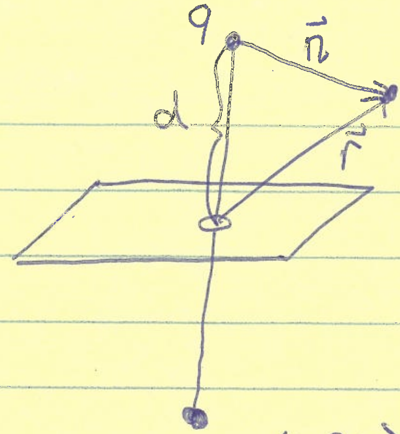
- satisfies Poisson's equation, → 0 @ ∞
- continuous at the boundary
- normal derivative is discontinuous at boundary

$$\sigma_b = -\epsilon_0\left(\frac{\partial V}{\partial z}\bigg|_{z=0^+} - \frac{\partial V}{\partial z}\bigg|_{z=0^-}\right) ; \sigma = -\epsilon_0\frac{\partial V}{\partial n}$$

$$= \frac{1}{2\pi}\left(\frac{\chi_e}{\chi_e+2}\right)\frac{qd}{(x^2+y^2+d^2)^{3/2}} \qquad = -\epsilon_0\frac{\partial V}{\partial z}\bigg|_{z=0}$$

- force on q

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q q_b}{(2d)^2}\hat{z} = \frac{-1}{4\pi\epsilon_0}\left(\frac{\chi_e}{\chi_e+2}\right)\frac{q^2}{4d^2}\hat{z}$$



$$q_b = \left(\frac{\chi_e}{\chi_e+2}\right) q$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{r}\,' = d\hat{z}$$

$$\vec{\mathscr{r}} = x\hat{x} + y\hat{y} + (z-d)\hat{z}$$

$$\Rightarrow |\mathscr{r}| = \left(x^2+y^2+(z-d)^2\right)^{1/2}$$

V is - continuous at boundary:

$$z=0: \quad V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{x^2+y^2+d^2} + \frac{q_b}{x^2+y^2+d^2}\right]$$

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q+q_b}{x^2+y^2+d^2}\right] \checkmark$$

- derivative

$$\left.\frac{\partial V}{\partial z}\right|_{z=0^+} = \frac{1}{4\pi\epsilon_0} q\left(-\frac{1}{2}\right)\left(x^2+y^2+(z-d)^2\right)^{-3/2} \cdot 2(z-d)$$

$$+ \frac{1}{4\pi\epsilon_0} q\left(-\frac{1}{2}\right)\left(x^2+y^2+(z+d)^2\right)^{-3/2} \cdot 2(z+d)$$

$$\vdots$$

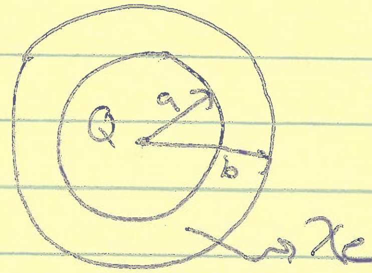
II.

Pg. 202

-polarization is built in

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E} + \epsilon_0 \chi_e \vec{E}$$
$$= \epsilon_0 (1+\chi_e)\vec{E}$$
$$\vec{D} = \epsilon \vec{E}$$

Problem 4.26:



$\oint \vec{D}\cdot d\vec{a} = Q_{f\,enc}$

- find the energy of configuration

$\vec{D} = \epsilon \vec{E}$

$$\vec{D} = \begin{cases} 0; & r<a \\ \dfrac{Q}{4\pi r^2}; & r>a \end{cases}$$

$$W = \frac{Q^2}{8\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\vec{E} = \begin{cases} 0; & r<a \\ \dfrac{Q}{4\pi\epsilon r^2}\hat{r}; & a<r<b \\ \dfrac{Q}{4\pi\epsilon_0 r^2}\hat{r}, & r>b \end{cases}$$

* electrostatic

$$\boxed{W = \frac{\epsilon_0}{2}\int E^2 d\tau}$$

-no dielectric (Prob. 2.34)

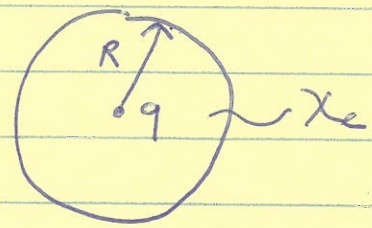
$$W = \frac{1}{2}\int \vec{D}\cdot\vec{E}\, d\tau = \frac{1}{2}\frac{Q^2}{(4\pi)^2}\cdot 4\pi\left\{\frac{1}{\epsilon}\int_a^b \frac{1}{r^2}\frac{1}{r^2} r^2 dr + \frac{1}{\epsilon_0}\int_b^\infty \frac{1}{r^2} dr\right\}$$

-derivation 4.4.3

$$= \frac{Q^2}{8\pi}\left\{\frac{1}{\epsilon}\left(\frac{-1}{r}\right)\Big|_a^b + \frac{1}{\epsilon_0}\left(\frac{-1}{r}\right)\Big|_b^\infty\right\}; \quad \epsilon_r = 1+\chi_e = \frac{\epsilon}{\epsilon_0}$$

$$\ldots \quad \frac{Q^2}{8\pi\epsilon_0(1+\chi_e)}\left(\frac{1}{a} + \frac{\chi_e}{b}\right)$$

I. Problem 4.35: Pg. 206



$\vec{D} = \epsilon \vec{E}$

$\epsilon = \epsilon_0(1+\chi_e)$

$$\oint \vec{D}\cdot d\vec{a} = Q_{enc,f} \Rightarrow \vec{D} = \frac{q}{4\pi r^2}\hat{r}$$

$$\vec{E} = \frac{1}{\epsilon}\vec{D} = \frac{q}{4\pi\epsilon r^2}\hat{r} = \frac{q}{4\pi\epsilon_0(1+\chi_e)r^2}\hat{r}$$

$$\vec{P} = \epsilon_0\chi_e\vec{E} = \frac{q\chi_e}{4\pi(1+\chi_e)r^2}\hat{r}$$

eq. 1.99

$$\rho_b = -\nabla\cdot\vec{P} = -\frac{q\chi_e}{4\pi(1+\chi_e)}\left(\nabla\cdot\frac{\hat{r}}{r^2}\right) = \frac{-q\chi_e\,\delta^3(\vec{r})}{1+\chi_e}$$

$$\sigma_b = \vec{P}\cdot\hat{r} = \frac{q\chi_e}{4\pi(1+\chi_e)R^2} \quad ; \quad r=R$$

- section 1.5

$$Q_{surf} = \sigma_b(4\pi R^2) = q\frac{\chi_e}{1+\chi_e}$$

$* \int \delta^3(\vec{r})\,d\tau = 1$

$$\int \rho_b\,d\tau = \frac{-q\chi_e}{1+\chi_e}\int \delta^3(\vec{r})\,d\tau = \frac{-q\chi_e}{1+\chi_e}$$

- at the center